

ESABASE2/Debris

Release 2.0

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I. Release Note

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IV. Glossary

Term	Description

V. List of Abbreviations

Abbreviation	Description
AU	Astronomical Unit (mean distance Sun – Earth)
ESA	European Space Agency
ESOC	European Space Operations Centre
ESTEC	European Space Research and Technology Centre
GEO	Geostationary Orbit
GTO	Geostationary Transfer Orbit
GUI	Graphical User Interface
ISS	International Space Station
LEO	Low Earth Orbit
LDEF	Long Duration Exposure Facility
MASTER	Meteoroid and Space Debris Terrestrial Environment Reference (Model)
MLI	Multi-layer Insulation
NASA	National Astronautic and Space Administration
ORDEM	Orbital Debris Engineering Model
RORSAT	Radar Ocean Reconnaissance Satellite

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Abbreviation	Description
SIMT	Study Information Management Tool
STS	Space Transportation System

VI. Distribution List

Company (Dept.)	Name	Comment
ESA/ESTEC	G. Drolshagen	as pdf via Email and SIMT

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1 Introduction

In this document, the physical models and technical background of the space debris and meteoroid environment modelling and risk analysis on which the enhanced ESABASE2/Debris software tool is built are described.

The software architecture and design itself is described in the design definition file /33/, the software handling in the software user manual /26/.

In Chapter 1 all debris and meteoroid models which have been implemented in the enhanced version of the ESABASE2/Debris software tool are described.

Four debris models are available within the ESABASE2/Debris simulation software:

- The NASA 90 model, which provides a simple and very fast debris flux calculation, but does not fully reflect the current knowledge of the Earth's debris environment, in particular the existence of a large number of particles on eccentric orbits. Additional shortcomings: the population is described by a small number of equations; the model is restricted to orbital altitudes below 1000km, and finally the age of the model.
- The NASA 96 model (also known as ORDEM 96) is the successor of the NASA 90 model and was implemented in former ESABASE/Debris versions. It is outdated and thus no longer included in ESABASE2/Debris.
- The MASTER 2001 model is based on numerical modelling of all known fragmentation events, SRM firings, NaK droplet releases, the Westford needles experiments, the generation of paint flakes by surface degradation effects, as well as the generation of ejecta particles and subsequent propagation of the particle orbits. The model provides realistic yearly population snapshots for the past and the future. The flux calculation is based on the analytic evaluation of the distributions of the size and the orbital elements of the particle population (MASTER 2001 Standard application). The model considers the population asymmetry induced by the asymmetric distribution of the particle orbits argument of perigee.
- The ORDEM2000 model describes the orbital debris environment in the low Earth orbit region between 200 and 2,000 km altitude. The model is appropriate for those engineering solutions requiring knowledge and estimates of the orbital debris environment (debris spatial density, flux, etc.). Incorporated in the model is a large set of observational data (both in-situ and ground-based), covering the object size range from 10 μm to 10 m and employing a new analytical technique utilizing a maximum likelihood estimator to convert observations into debris population probability distribution functions. These functions then form the basis of debris populations. OREDEM2000 uses a finite element model to process the debris populations to form the debris environment.
- The MASTER 2005 model is the successor of MASTER 2001. The model provides realistic four population snapshots per year for the past and the future. Compared to MASTER2001 lots of features have been significantly updated or added.

For meteoroids the omni-directional Grün model is maintained. It is described in this document.

Additionally, two further meteoroid models are implemented in ESABASE2/Debris, Divine-Staubach and MEM.

- The Divine-Staubach meteoroid model is part of the MASTER 2005 model. The model is based on the size and orbital element distributions of five meteoroid sub-populations, and thus provides directional information in the same way as the MASTER 2005 debris model.
- The MEM meteoroid model, developed by The University of Western Ontario, is a parametric model of the spatial distribution of sporadic meteoroids by taking their primary source to be short-period comets with aphelia less than 7 AU. It considers the contribution to the sporadic meteor complex from long-period comets and includes the effects of the gravitational shielding and focussing of the planets.

An enhanced stream model by Jenniskens, which is based on observation data gathered over a 10 year period, is available for the flux and damage analysis. This model includes directional information on the streams.

Further directional information is obtained by attempting to separate the β - meteoroids, which are driven away from the Sun into hyperbolic orbits by radiation pressure, from the α - meteoroids. An apex enhancement of the α - meteoroids and interstellar streams may introduce further directional information.

The meteoroid velocity and flux distribution according to Taylor is available in ESABASE2/Debris. This distribution is altitude enhanced for gravitational effects.

In the Chapter 3 the damage equations used in the software are described. A parametric approach has been chosen, allowing for flexibility in the usage of the damage equations. A new hole equation has been introduced, based on the latest research performed in this field at the University of Kent.

The behaviour of MLI as micro-particle debris shield was also investigated during the study. It was found that MLI can be characterised by the available parametric ballistic limit equations, either as single wall or multiple wall, depending on the analysis objectives.

In the Chapter 4, the Ejecta model is described. This feature of the ESABASE2/Debris software is based on a model developed by CERT/ONERA in Toulouse. The ejecta model has been updated, allows simulating the debris particle ejected from a primary impact with ray tracing.

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Finally, in Chapter 5 the techniques used for the damage and risk analysis using ray tracing technique is lined out. The new tool relies entirely on ray tracing for the computation of impact fluxes, failure fluxes and cratering fluxes. The ray tracing scheme which is implemented also allows accounting for Earth shielding and flux enhancements due to spacecraft motion (also known as the K factor). The full implementation of ray tracing allowed a smooth implementation of the enhanced directional effects of the environment models.

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2 Environment Models

This chapter describes the environment models which are implemented in the ESABASE2/Debris software. In this document, environment refers to the micro-particle environment of micro-meteoroids (natural particles) and space debris (man-made particles).

Due to their different characteristics, the two environments are presented separately.

2.1 The Space Debris Environment Models

2.1.1 Introduction

ESABASE/Debris, release 2 contained three debris flux models. While the *NASA 96* and the *MASTER 96 Hybrid model* provided flux results including directional information, the *NASA 90* model describes the debris environment by means of a set of analytical equations.

For release 3 of ESABASE/Debris the MASTER 96 model has been replaced by the *MASTER 2001 model*, which represents the state-of-the-art of debris modelling and offers some new features, which are available within ESABASE/Debris for the first time.

A major upgrade of the ESABASE software was performed in the framework of the “PC Version of Debris Impact Analysis Tool” contract. ESABASE/Debris was ported to the Windows PC platform. The ESABASE data model has been completely revised, a geometry modeller with basic CAD features was implemented and a state-of-the-art graphical user interface was developed. Additionally, NASA’s ORDEM2000 debris model was implemented. Due to the major changes and to distinguish between the Unix and the PC version of ESABASE, the PC version is called *ESABASE2*.

The following debris models are available in the latest release of ESABASE2/Debris:

NASA 90 Model (section 2.1.2)

This model has been the first more or less detailed description of the Earth's debris environment. It provides very fast, but less detailed debris flux analysis capabilities and is restricted to altitudes below 1000 km. The NASA 90 model has been maintained as an option and it is therefore briefly described in this document.

MASTER 2001 Model (Section 2.1.3)

The MASTER 2001 release (Ref. /23/) of the European MASTER model is based on a consequent upgrade and extension of the MASTER concept (Ref. /8/). The MASTER reference population as of May 1., 2001 now includes the population sources listed in Table 1:

Name	Origin	Particle size range
launch and mission related objects	all trackable objects except those generated by simulated fragmentation events such as explosions or collisions (corresponds to the catalogued objects/TLE background population of MASTER '99); includes the Westford needles, which were released during two American experiments (MIDAS 4 & 6) in the early sixties	0.5 mm ... 4 mm (Westford Needles) and 10 cm ... 10 m
fragments	resulting from and collisions	0.1 mm ... 10 m
NaK droplets	coolant droplets released by Russian RORSAT's	2 mm ... 4 cm
SRM slag particles	large particles released during the final phase of solid rocket motor firings	0.1 mm ... 3 cm
SRM Al ₂ O ₃ dust	small particles released during solid rocket motor firings	1 µm ... 80 µm
paint flakes	resulting from surface degradation	2 µm ... 0.2 mm
ejecta	resulting from meteoroid and debris impacts on exposed surfaces	1 µm ... 5 mm

Table 1 Population sources considered in the MASTER model

One of the most demanding aspects of the recent upgrade of the MASTER model is its capability to allow for flux and spatial density analysis for the complete space age, which is based on 3-monthly population snapshots. Moreover, three future debris population scenarios are provided by means of the corresponding yearly population snapshots (Ref. /23/). These future sub-populations include all particles larger than 1mm. Due to its large relevance for the future debris population evolution, the fragments are subdivided to explosion fragments and collision fragments.

Two flux analysis applications are offered by MASTER, the Analyst application and the Standard application. Since the database of the Analyst application is too big to be implemented into ESABASE2/Debris, the MASTER 2001 Standard application has been selected for the implementation.

ORDEM2000 Model (Section 2.1.4)

ORDEM2000 is the latest NASA debris engineering model and the successor of ORDEM96 (called NASA96 in ESABASE2/Debris). It is mainly based on measurement data

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originating from in-situ measurements, the examination of retrieved hardware and from ground based radar and optical observations. Auxiliary modelling with respect to the future space debris population was performed. The debris population data (spatial density, velocity distribution, inclination distribution) is provided by means of a so called Finite Element Model of the LEO Environment, and is provided by a set of pre-processed data files. For this purpose the region between 200- and 2000-km altitudes is divided into (5 deg × 5 deg × 50 km) cells in longitude, latitude, and altitude, respectively.

To calculate the flux on an orbiting spacecraft, the orbit of the spacecraft has to be specified. The model divides the orbit of the spacecraft into the specified number of segments in equal mean longitude (i.e., equal time) and then calculates the flux, from particles of six different sizes (10 μm to 1 m), on the spacecraft at each segment. The output results are stored in a flux table. It includes the altitude and latitude of the spacecraft at each segment and the fluxes from particles of six different sizes at that location. At the end of the table, fluxes averaged over the number of segments are given.

MASTER2005 Model (Section 2.1.5)

The latest release (Ref. /34/) of the European MASTER model is the successor of MASTER2001. Compared to the previous version, the following features have been significantly updated or added in the MASTER2005 release.

- Upgrade of the Debris Source Models
- Update of the Reference Population
- Unified Flux and Spatial Density Computation Concept
- Implementation of Damage Laws
- Flux and Spatial Density Analysis for Historic and Future Epochs

In difference to Master 2001 now only one unified analysis application is offered with Master 2005.

2.1.2 NASA 90 Model

2.1.2.1 NASA 90 Flux Model

The NASA 90 flux model, as published in Ref. /3/, was implemented in the original ESABASE/Debris software (Ref. /1/). Since this debris model is more efficient than the MASTER 2001 model with respect to the execution time, it remains a useful option in the enhanced ESABASE2/Debris software. For completeness the corresponding equations are here recorded again, using the nomenclature of Ref. /1/.

The flux F , which is the cumulative number of impacts on a spacecraft in a *circular orbit* per m² and year on a randomly tumbling surface is defined as a function of the minimum debris *diameter* d [cm], the target orbit *altitude* h [km] ($h \leq 1000$ km), the target

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orbit *inclination* i [degrees] , the *mission date* t [year] , and of the *solar radio flux* S (measured in the year prior to the mission).

$$R(d,h,i,t,S) = H(d) \Phi(h,S) \psi(i) [F_1(d) g_1(t,q) + F_2(d) g_2(t,p)]$$

$$H(d) = \left(10^{\exp\left[-(\log_{10} d - 0.78)^2 / 0.406\right]} \right)^{1/2}$$

$$F_1(d) = 1.22 \cdot 10^{-5} d^{-2.5}$$

$$F_2(d) = 8.1 \cdot 10^{10} (d+700)^{-6}$$

$$\Phi(h,S) = \Phi_i(h,S) (1 + \Phi_i(h,S))^{-1}$$

$$\Phi_i(h,S) = 10^{(h/200 - S/140 - 1.5)}$$

The functions $g_1(t,q)$ and $g_2(t,p)$ with the assumed annual growth rate of mass in orbit, p (default $p = 0.05$) and with the assumed growth rate of fragments q (default $q = 0.02$, and 0.04 after 2011) become

$$g_1(t,q) = (1 + q)^{t - 1988}$$

$$g_2(t,p) = 1 + p(t - 1988) .$$

In ESABASE2/Debris the population growth is accounted for linearly over the mission duration.

Finally the inclination dependent function $\psi(i)$ is tabulated as follows:

i [°]	28.5	30	40	50	60	70	80	90	100	120
$\psi(i)$	0.91	0.92	0.96	1.02	1.09	1.26	1.71	1.37	1.78	1.18

For intermediate values of i , a linear interpolation in $\psi(i)$ is performed.

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For the application of the ray tracing method to a fixed oriented plate the flux must be scaled by the cosine of the angle between the plate normal and the debris velocity arrival direction.

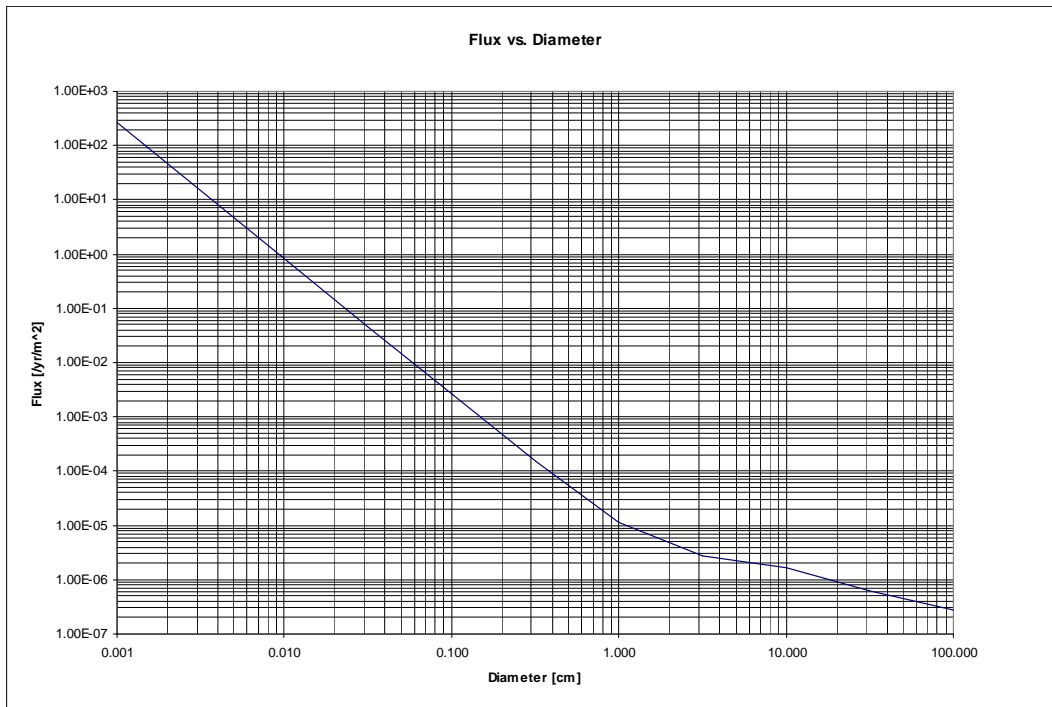


Figure 2-1 NASA 90 flux vs. diameter, 400 km / 51.6° orbit

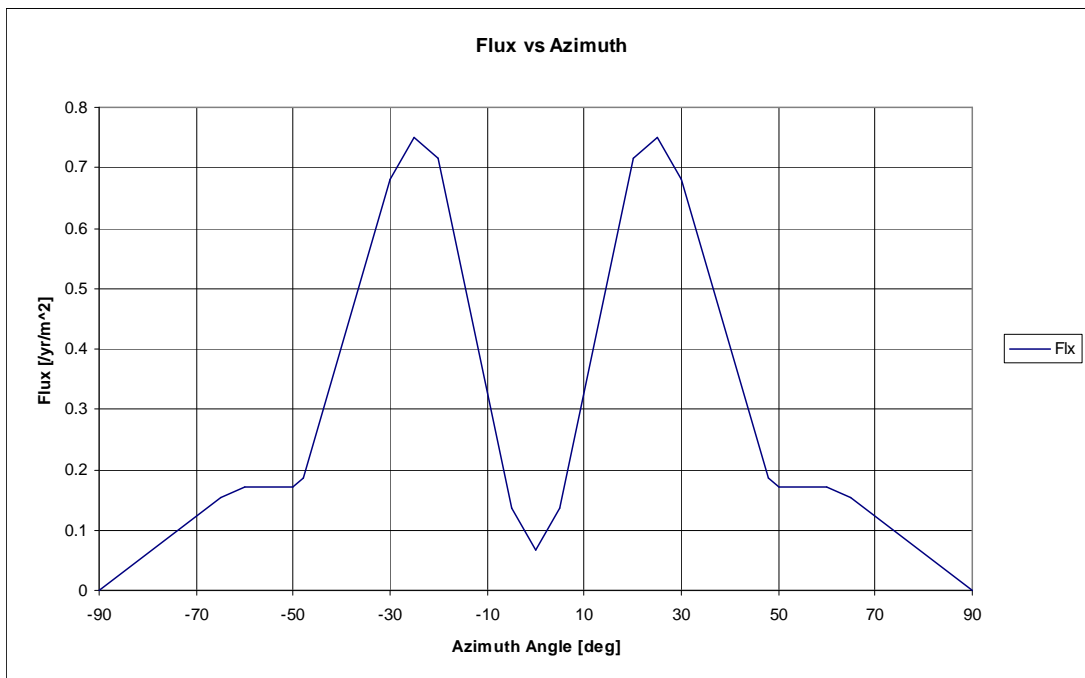


Figure 2-2 NASA 90 flux vs. azimuth, 400 km / 51.6° orbit

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2.1.2.2 NASA 90 Velocity Distribution

The collision velocity distribution $g(v)$ which represents the number of impacts with velocities between v and $v + dv$ is expressed as a function of inclination i in Ref. /3/. Taking into account that the orbital circular velocity at altitude h , $v_0(h)$ is occurring in the expression, it may be interpreted as a function of altitude. Thus, according to Ref. /1/, we may write

$$g(v, i, h) = v(2v_0 - v) \{g_1 \exp[-((v - 2.5v_0) / g_2 v_0)^2] + g_3 \exp[-((v - g_4 v_0) / g_5 v_0)^2]\} + g_6 v(4v_0 - v)$$

where the functions $g_1(i)$ to $g_6(i)$ are defined as follows, and $v_0(h)$ is the velocity at target orbit altitude h .

$$[G =] \quad g_1(i) = \begin{cases} 18.7 & i < 60^\circ \\ 18.7 + 0.0298 (i - 60)^3 & 60^\circ \leq i < 80^\circ \\ 250 & i \geq 80^\circ \end{cases}$$

$$[B =] \quad g_2(i) = \begin{cases} 0.5 & i < 60^\circ \\ 0.5 - 0.01 (i - 60) & 60^\circ \leq i < 80^\circ \\ 0.3 & i \geq 80^\circ \end{cases}$$

$$[F =] \quad g_3(i) = \begin{cases} 0.3 + 0.0008 (i - 50)^2 & i < 50^\circ \\ 0.3 - 0.01 (i - 50) & 50^\circ \leq i < 80^\circ \\ 0 & i \geq 80^\circ \end{cases}$$

$$[D =] \quad g_4(i) = 1.3 - 0.01 (i - 30)$$

$$[E =] \quad g_5(i) = 0.55 + 0.005 (i - 30)$$

$$[H.C =] g_6(i) = \begin{cases} 0.0125 (1 - 0.0000757 (i - 60)^2) & i < 100^\circ \\ [0.0125 + 0.00125(i - 100)] [1 - 0.0000757(i - 60)^2] & i \geq 100^\circ \end{cases}$$

$$v_0(i, h) = \begin{cases} v_0(h) \cdot (7.25 + 0.015 (i - 30)) / 7.7 & i < 60^\circ \\ v_0(h) & i \geq 60^\circ \end{cases}$$

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(The notations which are used in the original Ref. /3/ for the inclination dependent functions are listed in square brackets for comparison purposes.)

Since only circular orbits are represented by the NASA 90 model all debris are assumed to arrive in a plane tangent to the Earth. By vector addition one obtains for the direction dependence of the impact velocity v_{imp}

$$v_{imp} = 2 v_s \cos \alpha ,$$

where α is the angle between the satellite velocity vector and the debris arrival velocity vector. For a low Earth orbit $2 v_s$ is typically on the order of 15.4 km/s .

For the ray tracing method however, the debris velocity vector must be used and the impact velocity vector follows from numerical vector subtraction (see chapter 5).

2.1.2.3 Particle Mass Density

For the NASA 90 model the particle mass density can be either set to a constant with default value of $\rho = 2.8 \text{ g/cm}^3$ or the following dependency may be chosen:

$$\rho (d) = \frac{2.8}{d^{0.74}} \text{ [g/cm}^3\text{]} \quad \text{for } d \geq 0.62 \text{ cm}$$

$$\rho (d) = 4 \text{ g/cm}^3 \quad \text{for } d < 0.62 \text{ cm}$$

with d as the particle diameter.

It is suggested to use the same values for the NASA 96 model and for the MASTER 2001 model.

The density option as implemented in the software tool is identical for all debris models.

2.1.3 MASTER 2001 Model

2.1.3.1 Overview

As mentioned in section 2.1.1, the MASTER 2001 model has been implemented into ESABASE2/Debris by means of the Standard application.

The MASTER 2001 Standard application is an upgrade and extension of the MASTER '99 Standard application, which is described in detail in Ref. /20/. The approach is based on the mathematical theory used by N. Divine (Ref. /21/) to calculate meteoroid fluxes to detectors onboard probes in interplanetary space. After a thorough review, the theory has been adapted to spacecraft in Earth orbit.

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The population data describing the Earth's debris environment is derived from the MASTER reference population using comprehensive statistical analysis to "translate" the population given by representative objects to a population description by means of probability density distributions of the orbital elements and of the diameter and mass distributions (cf. Ref. /23/).

2.1.3.2 Flux Calculation

The basics of the Divine approach are described in Ref. /21/, /19/, and /23/. Although these descriptions of the model are well known and easily accessible, a short compilation of the most important equations is given in this section.

For the calculation of space debris flux to an Earth satellite, an Earth-centred equatorial co-ordinate system has to be used instead of the sun-centred ecliptic system. Furthermore, all focussing, shielding, and detector related factors (η_F , η_S , F_S , Γ) can be set to 1.

After the introduction of these changes, the flux on a target at a specified position on its orbit is derived from

$$J_M = \frac{1}{4} \sum_{dir=1}^4 [N_M \cdot (v_{imp})_{dir}] \quad (1)$$

where N_M is the spatial density

$$N_M = \frac{H_M}{\pi} \cdot \int_0^{\pi/2} N_1(\sin \chi) d\chi \cdot \int_{e_\chi}^1 \frac{P_e}{\sqrt{e - e_\chi}} de \cdot \int_{|\delta|}^{\pi - |\delta|} \frac{(\sin i) p_i}{\sqrt{\cos^2 \delta - \cos^2 i}} di \quad (2)$$

The impact velocity v_{imp} is the velocity difference

$$v_{imp} = \left| \vec{v}_{part} - \vec{v}_{tar} \right| , \quad (3)$$

and the cumulative size distribution including the number of particles of the specified population is

$$H_M = \int_m^\infty H_m dm \quad (4)$$

N_1 , p_e and p_i are the differential distributions of the orbital elements of the particles. Integration over these distributions, using the auxiliary variable χ (s. /18/, equation (7)), gives the spatial density for all particles whose size exceeds the lower size threshold m . If the integration in equation (4) is carried out over a certain size range it gives the

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number of particles in this size range and thus flux or spatial density for this size range is evaluated. The limits of the integrals in equation (2) ensure, that only particle orbits are considered, which may reach the target position:

- The particle orbit perigee altitude has to be below the altitude of the target at its current position. This requirement is considered in the first integration (over the perigee radius distribution).
- The eccentricity of the particle's orbit must exceed a minimum value e_{χ} (s. /18/, equation (7)) to be able to reach the target. This is considered in the second integration (over the eccentricity distribution).
- The particle orbit inclination i has to be equal or larger than the declination $|\delta|$ of the target position, and less than or equal to $|180^{\circ} - \delta|$. This is considered in the third integration (over the inclination distribution).

The summation in equation (1) takes into account, that due to the assumption of uniform distributions of the particle's right ascension of ascending node and argument of perigee four velocity directions are possible with the same probability.

In order to obtain correct results it became necessary to use so called 'textbook' distributions to describe the particles orbital elements (refer to /19/). Those 'textbook' distributions are probability density functions, which has to be transformed to the distributions used by Divine using the transformations given in Table 2:

distribution	symbol of 'textbook' distribution	condition for 'textbook' distribution	transformation
perigee radius R_1	D_1	$\int_0^{\infty} D_1(r_1) dr_1 = 1$	$N_1 = \frac{D_1}{r_1^2}$
eccentricity e	D_e	$\int_0^1 D_e(e) de = 1$	$p_e = (1-e)^{\frac{3}{2}} D_e$
inclination i	D_i	$\int_0^{\pi} D_i(i) di = 1$	$p_i = \frac{D_i}{2\pi^2 \sin i}$

Table 2 Transformation from 'textbook' distributions to Divine's distributions

Some assumptions in the theory of the selected approach require a certain effort to make this solution applicable to the needs of a debris model:

- Different distributions of the orbital elements of particles of different size within one population (e.g. fragments) do not allow describing the population by only four distributions (mass or diameter, perigee radius, eccentricity, inclination).

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- Cross-coupling effects between the orbital elements of the particles are not considered in the approach. This may lead to the calculation of flux contribution from objects, which are not existing in reality (e.g. in the SRM slag population).
- The assumption of symmetric particle distribution with respect to the equatorial plane and with respect to the Earth's rotation axis may result in an inaccurate description of some source populations, namely those, which do not fulfil the symmetry assumption (e.g. parts of the catalogued objects population, such as Molniya-type orbits).

These problems have been solved during the development of the MASTER 2001 Standard application:

- A population pre-processing tool – called PCube – has been developed, which automatically creates the Standard application population input files. The generation of the size and orbital element distributions is based on a comprehensive statistical analysis of the populations. So called cross-coupling effects between the size distribution and the orbital element distributions on one hand, and between the orbital element distributions on the other hand are identified using the statistical method of a cluster analysis.
- The approach to consider asymmetries in the population is based on the fact, that each of the four possible impact velocities (in case of population symmetry) can be related to a well defined particle nodal line position and perigee position. Thus, each impact velocity – and consequently flux value – may be "weighted" with a factor related to the distribution values of the right ascension of ascending node distribution and the argument of perigee distribution. Within ESABASE2/Debris, the described asymmetries are considered as follows:
 - right ascension of ascending node: Off for all sub-populations,
 - argument of perigee: On for all sub-populations except the SRM dust sub-population.

The results of the new Standard application have been verified against the reference results of the MASTER Analyst application (Ref. /23/).

2.1.3.3 Population Snapshots

The MASTER 2001 model provides realistic historic population snapshots from the beginning of spaceflight in 1957 until the reference epoch May 1st, 2001. Additionally, three different future population snapshots for each year from 2002 until 2050 are provided under the assumption of three different debris environment evolution scenarios.

Within the ESABASE2/Debris implementation of MASTER 2001, the following sub-sets of these population snapshots are available:

Historic populations from 1980 to 2001, one snapshot per year.

Future populations from 2002 to 2020, one snapshot per year, reference scenario (no future constellations, no mitigation, continuation of recent traffic).

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Due to the fact, that ESABASE2/Debris does not contain a time loop, but considers population evolution during the mission duration by applying a population growth rate which is specified by the user, the debris analyser makes use of the population snapshot which is closest to the mission start epoch. The population growth factor is not considered, if the debris flux is calculated with the MASTER 2001 model.

If it is intended to analyse the debris risk as a function of time, subsequent ESABASE2/Debris runs have to be performed with different analysis time start epochs.

2.1.3.4 Results

This section provides a brief description of the MASTER 2001 model results, which are used for flux calculation and damage assessment within ESABASE2/Debris.

Four two-dimensional spectra, and one three-dimensional spectrum are generated by the model. The spectra definitions are given in Table 3:

Spectrum	min. value	max. value	number of steps
flux vs. diameter	as specified for the analysis		32
flux vs. impact velocity	0 km/s	40 km/s	80
flux vs. impact azimuth angle	-180°	180°	90
flux vs. impact elevation angle	-90°	90°	90
flux vs. impact velocity and impact azimuth angle	as specified for the corresponding 2D spectra		

Table 3 MASTER 2001 flux spectra

Figure 2-3 to Figure 2-7 provide the results (cross-sectional flux on a sphere) of the MASTER model for an ISS-like orbit. The diameter spectrum (Figure 2-3) is given for the complete size range of the MASTER model, while the other spectra are given for a lower diameter threshold of 0.1mm.

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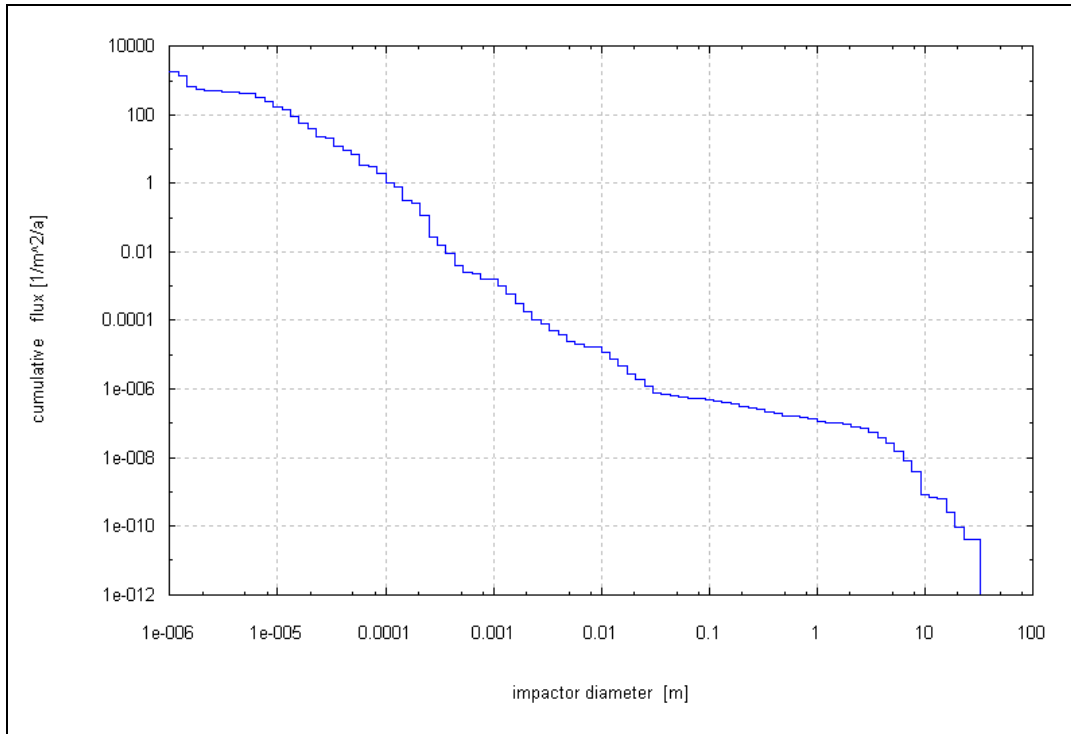


Figure 2-3 MASTER 2001 flux vs. particle diameter, 400 km / 51.6° orbit

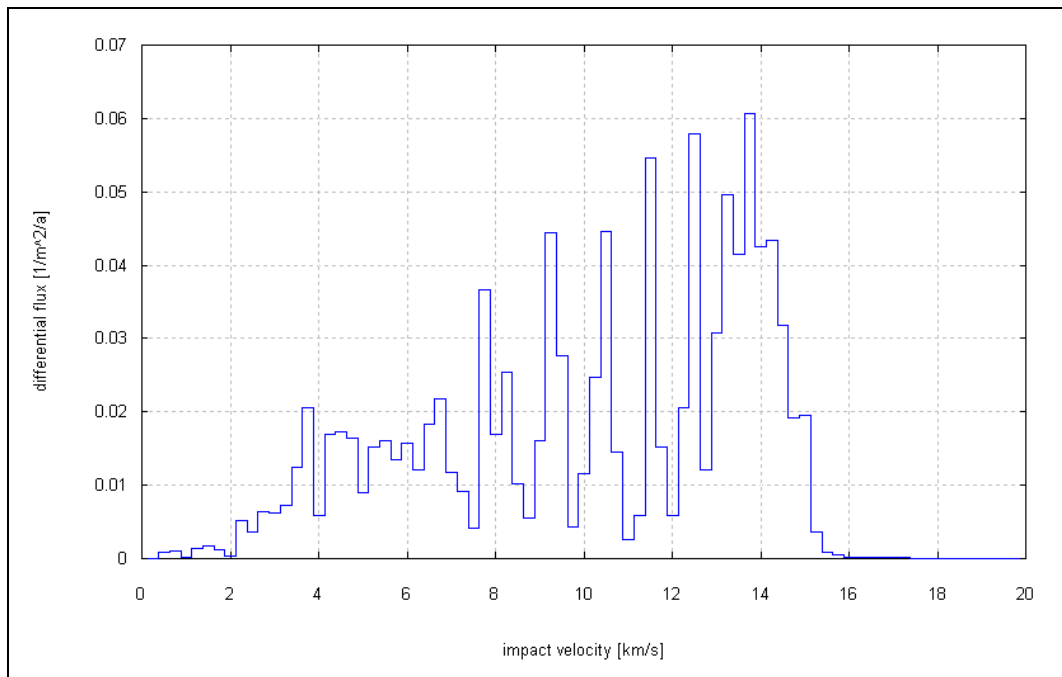


Figure 2-4 MASTER 2001 flux vs. impact velocity, 400 km / 51.6° orbit / d > 0.1mm

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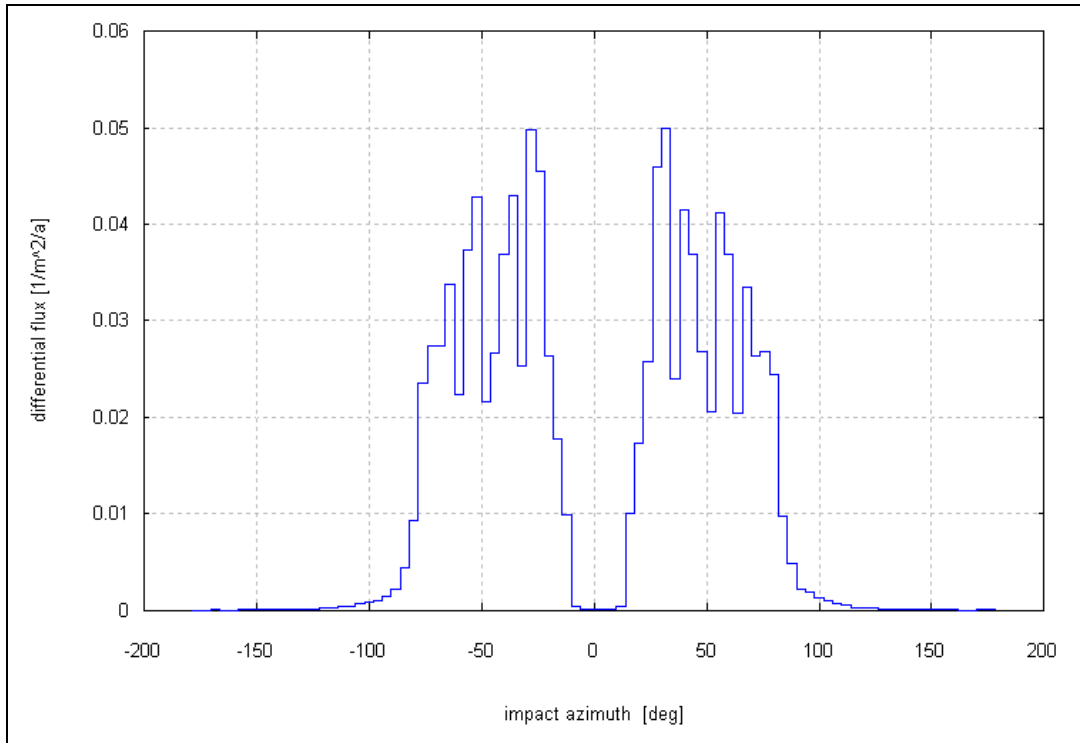


Figure 2-5 MASTER 2001 flux vs. azimuth, 400 km / 51.6° orbit / d > 0.1mm

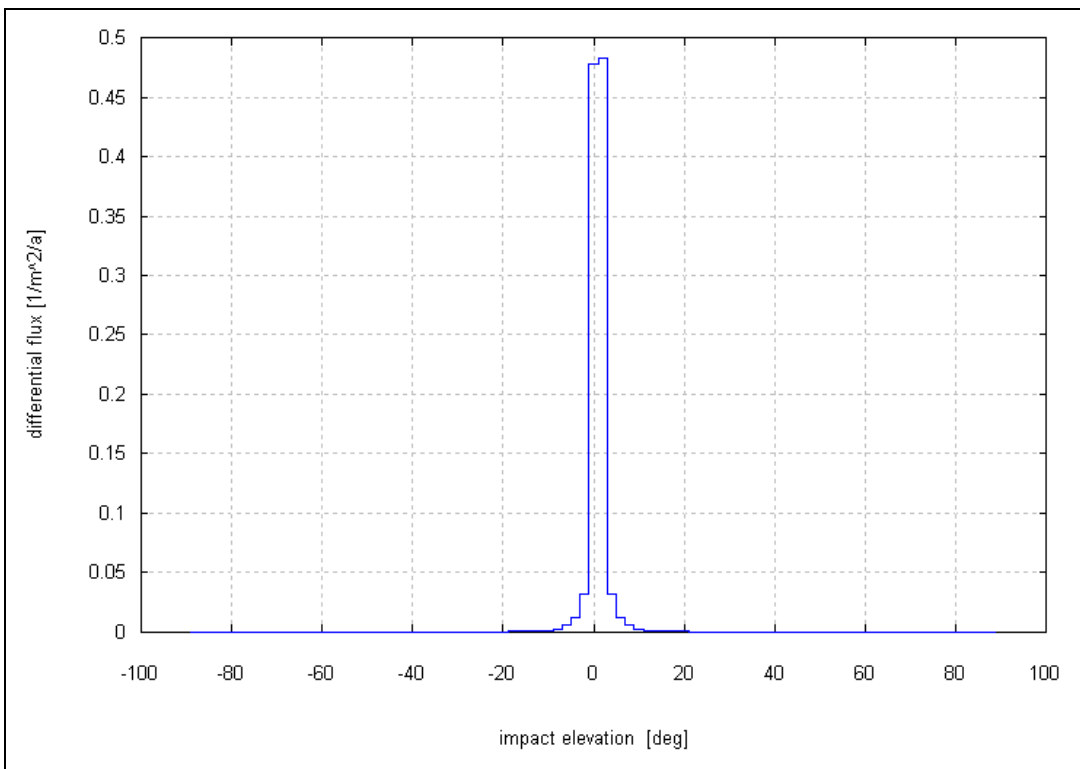


Figure 2-6 MASTER 2001 flux vs. elevation, 400 km / 51.6° orbit / d > 0.1mm

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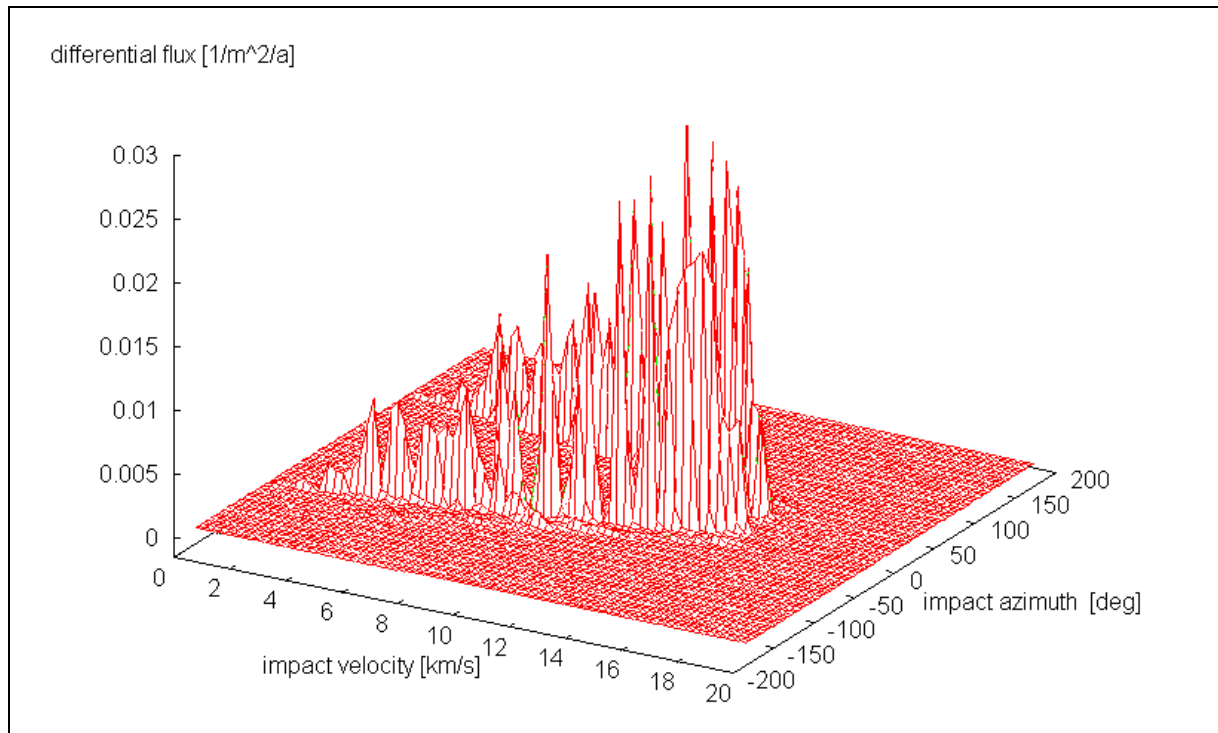


Figure 2-7 MASTER 2001 flux vs. velocity and azimuth, 400 km / 51.6° orbit / d > 0.1mm

Although the spectra are displayed as differential distributions – except the diameter spectrum, which is cumulative – for compatibility with the results of the other debris models (see section 2.1.2.1), the distributions are provided and used in their cumulative form within the ESABASE2 analysis as described in chapter 5.

In ESABASE2/Debris the MASTER 2001 flux analysis is performed for single orbital points specified by the user. This differs from the previous ESABASE version, where the flux analysis was performed for orbital arcs centred around each orbital point so that the entire orbit is covered. This change might result in partly considerable difference in the analysis results. It became necessary to change the implementation to yield results comparable to those of ORDEM2000, where flux is always related to single orbital points instead of orbital arcs.

2.1.4 ORDEM2000 Model

2.1.4.1 Overview

With the establishment of the ORDEM2000 engineering model NASA implemented a completely different approach compared to the NASA90 and ORDEM96 (NASA96) models. Here, the debris population is described by the distributions of spatial density and

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velocity in space. Figure 2-8 outlines the different approaches of ORDEM2000 and ORDEM96.

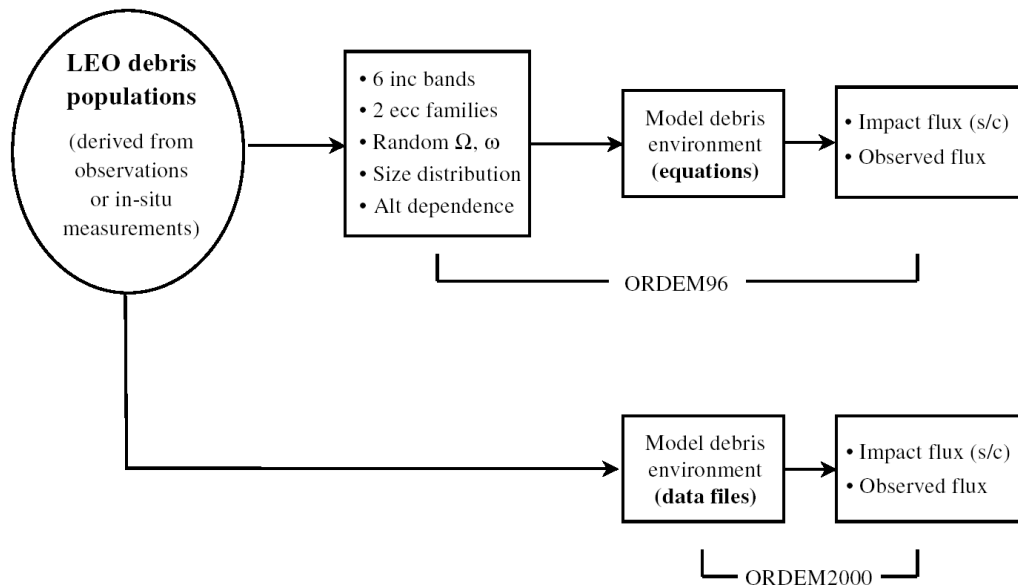


Figure 2-8 Comparison of the approaches of ORDEM2000 and ORDEM96 /27/

Once a debris population is derived from existing data, ORDEM96 simplifies the population into 6 inclination bands and 2 eccentricity families /7/. Objects within each inclination band are assumed to have the same inclination rather than a distribution of inclinations. The ORDEM2000 debris environment model describes the spatial density, velocity distribution, and inclination distribution of debris particles at different latitudes and altitudes. The debris environment is represented by a set of pre-processed data files. No assumptions regarding debris particles' inclinations, eccentricities, or orientations in space (longitudes of the ascending node and arguments of perigee) are required in this approach. However, ORDEM2000 uses a randomized distribution of the objects' right ascension of the ascending nodes.

2.1.4.2 Observation Data Sources and Modelling Approach

Table 4 represents a list of all observation data sources used in the establishment of the ORDEM2000 model. A detailed description of the data sources, processing and analysis can be found in /27/.

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	Size range	Altitude Range (km)	Inc. Range (degree)	Time of Collection
SSN	10 cm to 10 m	200 to 2000	All	up to Dec. 99
Haystack	0.3 cm to 10 m 0.5 cm to 10 m 0.5 cm to 10 m 0.5 cm to 10 m 1.0 cm to 10 m	350 to 1100 350 to 650 350 to 650 700 to 1100 1200 to 2100	40 to 140 28 to 152 32 to 148 32 to 148 40 to 140	91 to 99 91 to 99 91 to 94 94 to 98 93,94,96,97
HAX	1.0 cm to 10 m 0.8 cm to 10 m	450 to 1050 450 to 1050	40 to 140 40 to 140	94 to 97 98 to 99
LDEF^a	0.01 to 1 mm	330 to 480	All	Apr. 84 to Jan. 90
HST-SA	0.01 to 1 mm	586 to 614	All	Apr. 90 to Dec. 93
EuReCa	0.005 to 0.5 mm	502 to 508	All	Aug. 92 to Jun. 93
Shuttle^b	0.1 to 1 mm	300 to 400	All	95 to 98
SFU	10 µm to 1 mm	480	All	Mar. 95 to Jan. 96
Mir	10 to 100 µm	170 to 300	All	Mar. 96 to Oct. 97
Goldstone	2 mm to 2 cm	280 to 2000	32 to 148	Oct. 94 to Oct. 98

^aLDEF: Space Debris Impact Experiment (/30/, /31/), Chemistry of Meteoroid Experiment (/28/, /29/), Interplanetary Dust Experiment (F. Singer), LDEF frame (M/D Special Investigation Group).

^bShuttle: STS-50, 56, 71, 72, 73, 75, 76, 77, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 94, 95, 96.

Table 4 Data sources used in the establishment of ORDEM2000 /27/

The ORDEM2000 model is based on five pre-calculated debris populations. They correspond to objects of five different size thresholds: 10 µm and greater, 100 µm and greater, 1 cm and greater, 10 cm and greater, and 1 m and greater (hereafter referred to as 10-µm, 100-µm, 1-cm, 10-cm, and 1-m populations). The major sources

- SSN catalog (build the 1-m and 10-cm populations),
- Haystack radar data (build the 1-cm population),

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- LDEF measurements (build the 10- μm and 100- μm populations),

were used to build the debris populations, while the other sources were used to verify and validate the model predictions.

Since no direct measurement at 1 mm is available, the 1-mm debris population in the model is based on an interpolation between the 100- μm and 1-cm populations. Goldstone radar data for the 3-mm objects are used to justify the interpolation.

The reference date for the debris populations was selected to be January 1, 1999. The SSN catalogue from the same reference date was used, and the Haystack debris detection from each year was projected to the reference date using the historical growth rate of the 1-cm population from the NASA orbital debris evolution model EVOLVE 4.0 (/32/). Then, the combined Haystack data was used to build the 1-cm population as of January 1, 1999. The LDEF debris impact data are first processed with a simple model that calculates the historical 10- μm and 100- μm debris populations, including the effects of atmospheric drag and solar radiation pressure. Then, the number of debris impacts detected during the LDEF mission (1984-1990) was scaled with the model prediction during the same period, and then projected to January 1, 1999.

2.1.4.3 The LEO Debris Environment Model

Figure 2-9 shows the subdivision of the region between 200 km and 2000 km altitude into 5 deg \times 5 deg \times 50 km cells in longitude (θ), latitude (90 deg- φ), and radius (r), respectively. The resident time of each (observed) debris particle within each cell is calculated using the fractional time that it spends in that cell. For example, if a debris particle spends 3% of its orbital period within a given cell, 0.03 "object" is assigned to that cell. Once the same procedure is completed for every debris particle in the population, the spatial density of this debris population within each cell is simply the sum of objects within that cell divided by its volume V_{cell} , where

$$V_{cell} = \iiint r^2 (\sin \varphi) dr d\varphi d\theta,$$

and r , φ , and θ are defined in Figure 2-9.

The velocity of a debris particle within a given cell is calculated in two steps. The first step is to convert its orbital elements to the velocity and position vectors in the geocentric equatorial system. The second step is to transfer the velocity components to a special local system via two coordinate transformations. The local system is a right-handed geocentric system where the x-axis points in the radial-outward direction, the y-axis points in the local east direction, and the z-axis points in the local north direction. The plane defined by the y-axis and z-axis is the local horizontal.

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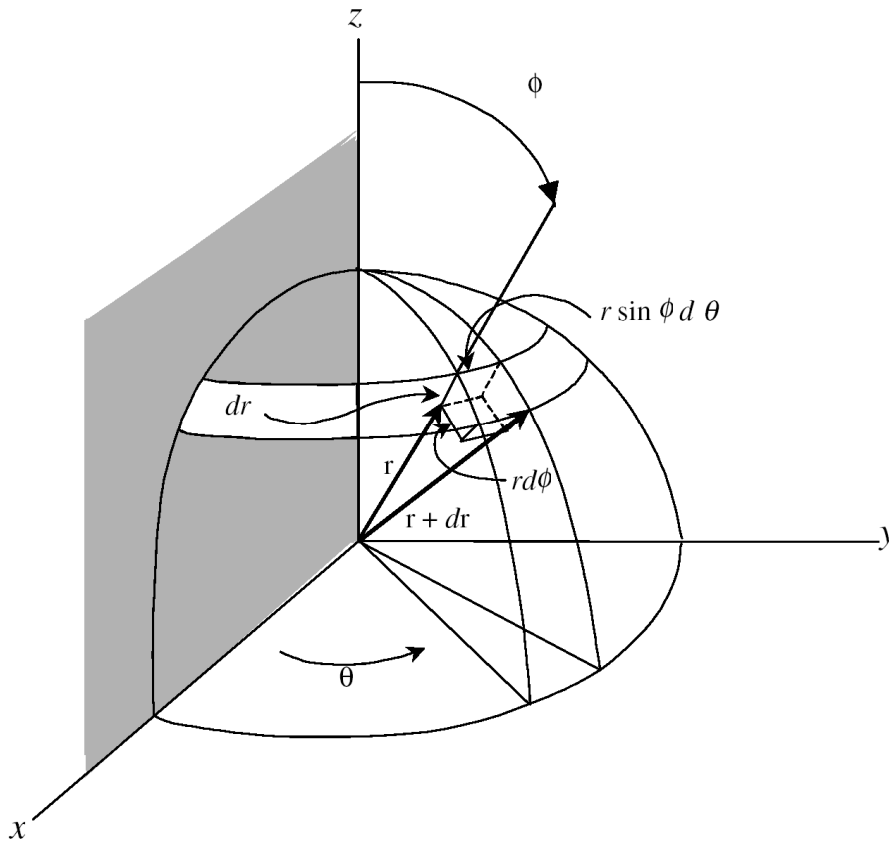


Figure 2-9 Definition of the cells /27/

Let (v_x, v_y, v_z) be the geocentric equatorial velocity components of a debris particle in a given cell. The components (v_{x2}, v_{y2}, v_{z2}) in the local system are calculated with the following two transformations:

$$v_{x1} = v_x \cos\theta + v_y \sin\theta$$

$$v_{y1} = -v_x \sin\theta + v_y \cos\theta$$

$$v_{z1} = v_z$$

and

$$v_{x2} = v_{x1} \cos(90^\circ - \phi) + v_{z1} \sin(90^\circ - \phi)$$

$$v_{y2} = v_{y1}$$

$$v_{z2} = -v_{x1} \sin(90^\circ - \phi) + v_{z1} \cos(90^\circ - \phi),$$

where θ and ϕ are defined in Figure 2-9.

The velocity distribution of debris particles within a given cell is calculated using all particles in the cell, weighted by their individual spatial densities. To reduce the size of the

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templates, only the velocity components in the local horizontal plane are recorded. This is justified since the radial velocity component is generally less than 0.1 km/s while the horizontal velocity component is about 6 km/s to 11 km/s. The velocity distribution within each cell is stored in a magnitude-and-direction two-dimensional matrix, as shown in Figure 2-10. The magnitude ranges from 6 km/s to 11 km/s with an increment of 1 km/s while the direction ranges from 0 deg to 360 deg with an increment of 10 deg. Each element in the matrix gives the fraction of particles with a velocity within the magnitude and direction specified by the position of the element. For example, the 2% element in Figure 2-10 indicates that 2% of all particles in this three-dimensional cell have their orbital velocity (in the local horizontal plane) between 6 km/s and 7 km/s with a direction between the local east and 10 deg northward. The sum of all elements in a matrix is always 100%.

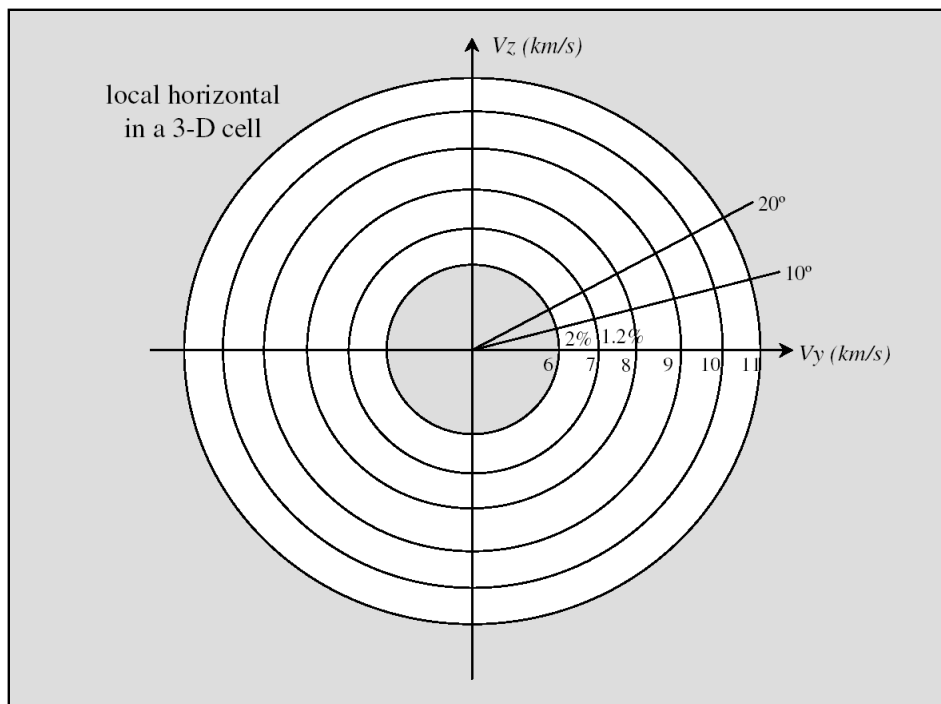


Figure 2-10 Velocity distribution matrix /27/

The inclination distribution of debris particles within each cell is also calculated and saved as part of the template files. The range is between 0 deg and 180 deg with an increment of 2 deg.

2.1.4.4 Results

Some exemplary results of ORDEM2000 are displayed in Figure 2-11 to Figure 2-13. Since the three-dimensional velocity vs. impact angle distribution of ORDEM2000 provides the percentage of debris objects coming from a particular direction with a particular velocity, and ESABASE2 requires the flux vs. impact velocity and impact azimuth angle distribution, the latter distributions have to be derived from the ORDEM2000 results. ORDEM2000 generates the 3D output for each analysed orbital point. Due to the fact

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that the flux vs. diameter distributions are also given for each orbital point, these can be considered in the generation of the 3D flux vs. velocity and azimuth distribution used by ESABASE2.

Figure 2-11 gives the average flux vs. diameter. While ORDEM2000 performs a cubic spline interpolation, ESABASE2 interpolates linearly. This leads to differences in the 100 μm to 1 mm and in the 1 cm to 10 cm diameter ranges. However, these differences will become visible in the ESABASE2 output only, if the user selects a lower diameter threshold within the named diameter ranges, e.g. 300 μm or 2 cm.

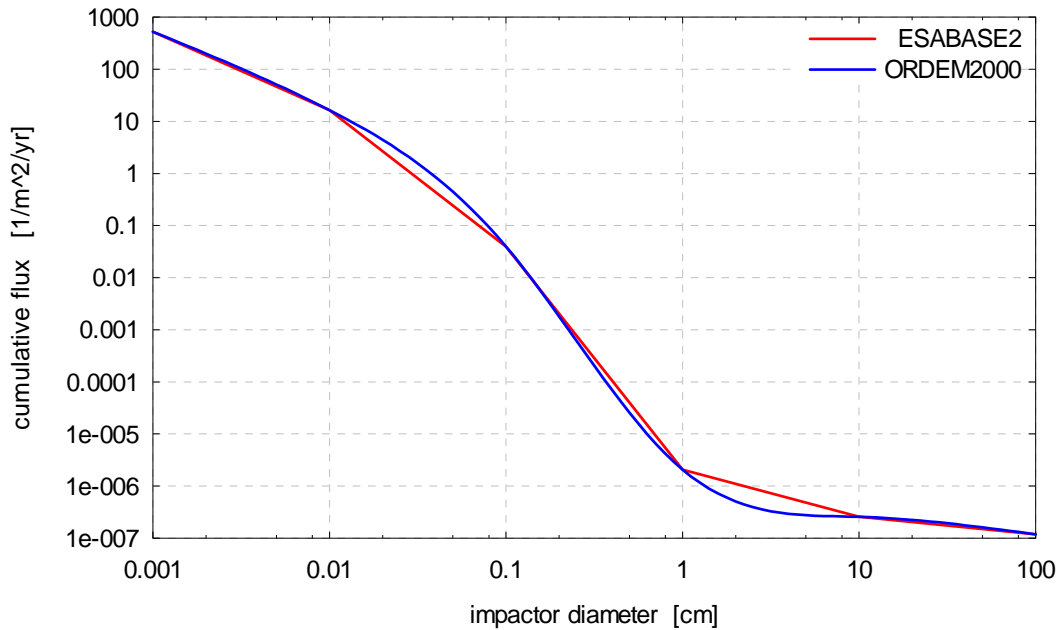


Figure 2-11 ORDEM2000 flux vs. diameter, ISS-like orbit

Figure 2-12a shows the relative flux vs. impact angle distribution, where the “impact” angle is not related to the spacecraft orbit or orientation, but to the horizontal plane of the cell corresponding to the spacecraft position (cp. Figure 2-10). 0 deg is the East direction, 90 deg North and so on.

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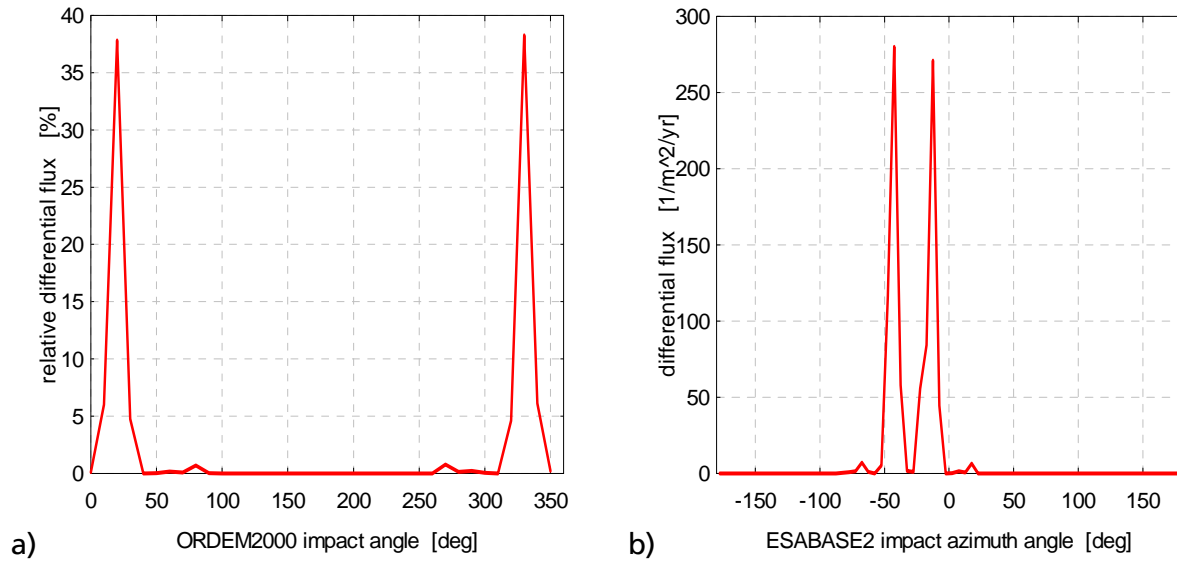


Figure 2-12 a) ORDEM2000 flux vs. impact angle, b) corresponding ESABASE2 flux vs. impact azimuth angle, ISS-like orbit at the ascending node, $d > 10 \mu\text{m}$

Consequently, the distribution given in Figure 2-12a has to be translated to an impact azimuth angle distribution which is used by ESABASE2 to derive the random ray directions. Figure 2-12b shows the impact azimuth angle distribution calculated from the ORDEM2000 impact angle distribution for a given size class (particle diameter: $10 \mu\text{m}$). The “translation” has to be performed under consideration of the particle velocity distribution. The azimuth angle is the angle between the projection of the impact velocity vector to the local horizontal plane and the space craft velocity vector. It is positive if the particle arrives from the left side.

One can see that the peaks of the almost symmetric ORDEM2000 distribution are reflected in the azimuth distribution. This is underlined by Figure 2-13a and b: The peaks can be found in both distributions.

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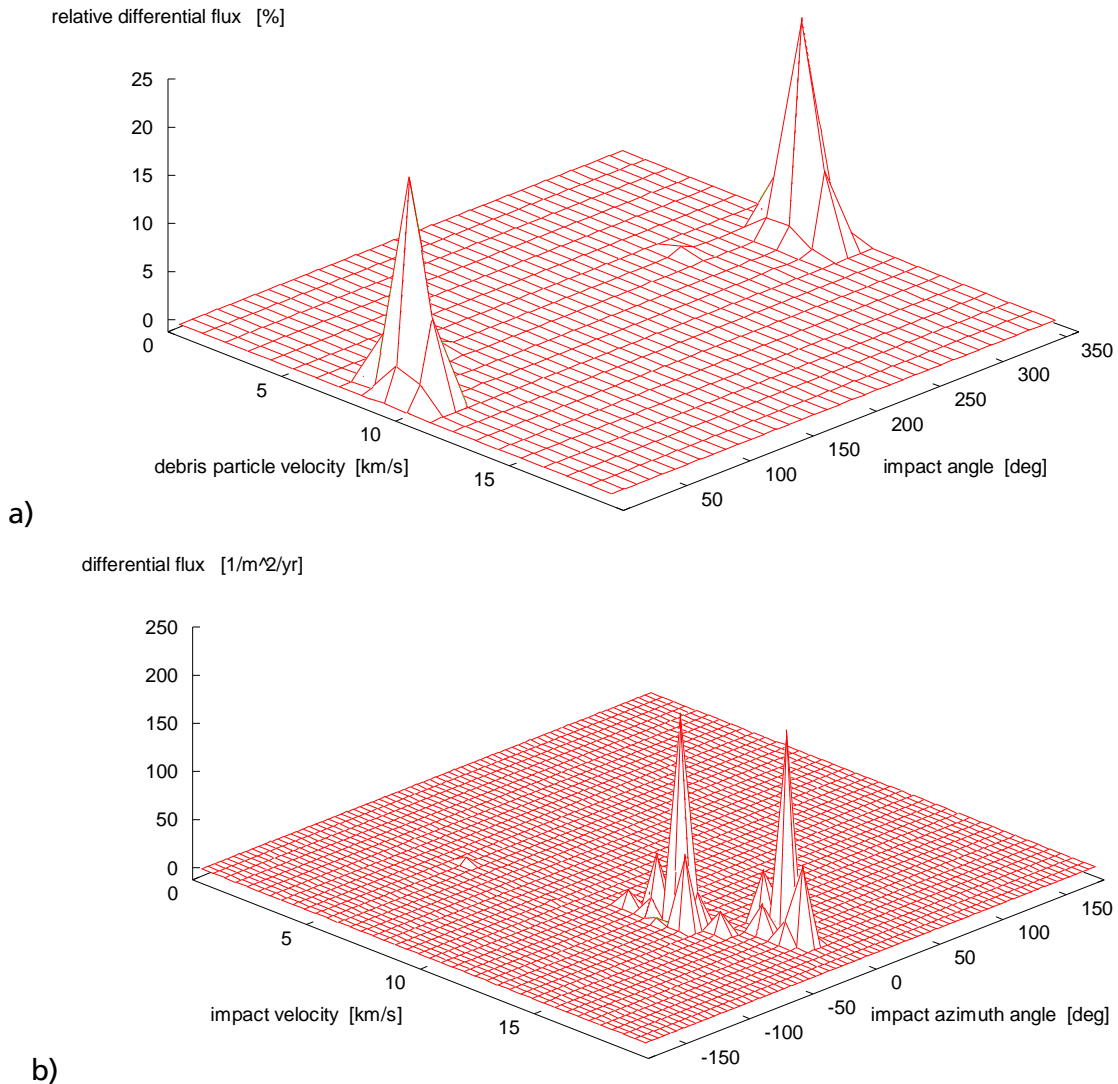


Figure 2-13 a) ORDEM2000 flux vs. debris particle velocity and impact angle, b) corresponding ESABASE2 flux vs. impact velocity and impact azimuth angle, ISS-like orbit at the ascending node, $d > 10 \mu\text{m}$

However, the almost symmetric distribution shown in Figure 2-13a becomes asymmetric when transferred to Figure 2-13b. This asymmetry is a consequence of the consideration of the particle and the spacecraft velocity vectors.

2.1.4.5 Limitations

The applicability of ORDEM2000 within the ESABASE2/Debris application is limited by the following facts:

- The altitude range of ORDEM2000 is 200 km to 2000 km. Consequently, all orbits with higher orbital altitude (also in parts of the orbit, e.g. GTO) cannot be ana-

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lysed with ORDEM2000. The MASTER 2001 model is currently the only debris model which allows the analysis of orbits up to 1000 km above GEO.

- ORDEM2000 includes debris particles in the size range from 10 μm to 1 m.

Eccentric debris particle orbits are not considered in the determination of the impact direction (velocity component in the local horizontal plane only), i.e. ORDEM2000 does not provide an impact elevation angle distribution. Consequently, similar to NASA90, no flux will be calculated on surfaces which are parallel to the local horizontal plane.

2.1.5 MASTER2005 Model

2.1.5.1 Overview

Upgrade of the Debris Source Models

The following debris source models have been upgraded in MASTER2005:

- The NASA break-up model has been revised for object sizes smaller 1 mm with a re-definition of the area-to-mass distribution and an increase of the delta velocity distribution.
- The size distribution parameter settings for SRM slag and dust, paint flakes, and ejecta have been revised based on newly available impact measurement data.
- The NaK droplet model is based on a physical description of the release mechanism. This includes new size, velocity, and directional distributions.
- The ejecta model has been thoroughly reviewed which results in major changes to the orbital distribution compared to the former MASTER release.
- The release model for surface degradation products (paint flakes) now depends on the changing atomic oxygen density environment near Earth due to the solar activity.

Update of the Reference Population

The processing of debris generation mechanisms (SRM firings, fragmentations, NaK release events, etc.) were considered and the resulting population propagated to the new reference epoch of May 1, 2005. The updated list of events now comprises 203 fragmentations, 1076 SRM firings, 16 NaK droplet releases, and 2 West Ford needle deployments. The update also includes processing of the ongoing generation of surface degradation products and ejecta.

Unified Flux and Spatial Density Computation Concept

The MASTER2001 high precision flux prediction tool ANALYST was upgraded to provide the user with spatial density computations. This new MASTER application is the only flux browser on the user side of MASTER2005. It combines a quick assessment of spatial density characteristics with high resolution flux results. The statistical flux determination

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approach based on probability tables for the object characteristics is now used for all debris sources.

Flux and Spatial Density Analysis for Historic and Future Epochs

The storage needs for the probability tables enables the generation of population snapshots for the complete space age on a single DVD, ranging from 1957 to the future (2055). For future epochs the user may select between three different population evolution scenarios. The stand-alone version of MASTER 2005 allows a flux and spatial density analysis for any epoch within the mentioned time span. However, the user should be aware of the computation time, which may drastically increase subject to the analysis parameter settings (time interval, target orbit, number of populations to be considered, number of spectra to be generated, etc.).

2.1.5.2 Observation Data Sources

The debris environment of the Earth provided with MASTER 2005 contains different sources down to a particle diameter of 1 μm . Figure 2-14 shows the different debris sources and its corresponding size range.

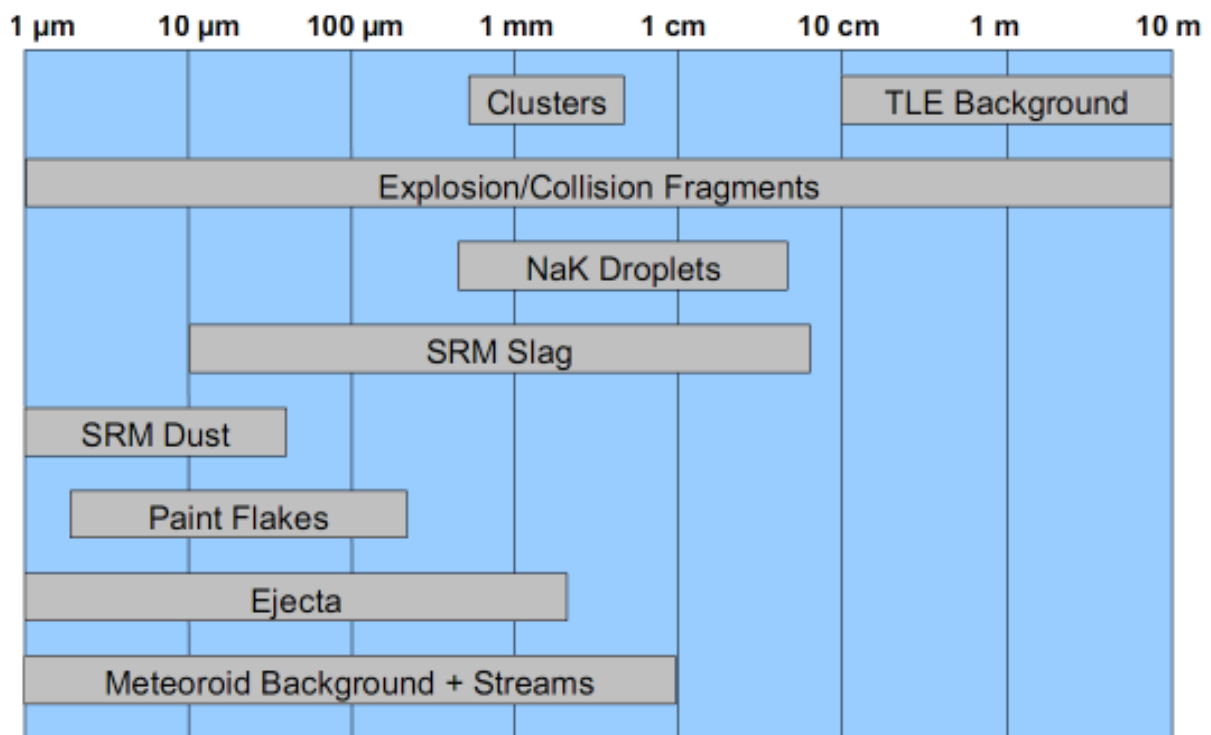


Figure 2-14: Debris and meteoroid sources considered in MASTER 2005 model

The MASTER 2005 model provides realistic historic population snapshots (3-monthly) from the beginning of spaceflight in 1957 until the reference epoch May 1st, 2005. Additionally, three different future population snapshots for each year from 2006 until 2055 are provided under the assumption of three different debris environment evolution scenarios. Further details can be found in /34/.

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Within the ESABASE2/Debris implementation of MASTER 2005, the following sub-sets of these population snapshots are available:

Historic populations from 1980 to 2005, one snapshot (May 1st) per year.

Future populations from 2006 to 2020, one snapshot per year, reference scenario (no future constellations, no mitigation, continuation of recent traffic).

As for MASTER 2001 the debris analyser makes use of the population snapshot which is closest to the mission start epoch. The user specified population growth factor is not considered, if the debris flux is calculated with the MASTER 2005 model.

If it is intended to analyse the debris risk as a function of time, subsequent ESABASE2/Debris runs have to be performed with different analysis time start epochs.

2.1.5.3 Results

This section provides a brief description of the MASTER 2005 model results, which are used for flux calculation and damage assessment within ESABASE2/Debris.

Four two-dimensional spectra, and one three-dimensional spectrum are generated by the model. The spectra definitions are given in Table 3:

Spectrum	min. value	max. value	number of steps
flux vs. diameter	as specified for the analysis		≤ 32
flux vs. impact velocity	0 km/s	40 km/s	80
flux vs. impact azimuth angle	-180°	180°	90
flux vs. impact elevation angle	-90°	90°	90
flux vs. impact velocity and impact azimuth angle	as specified for the corresponding 2D spectra		

Table 5 MASTER 2005 flux spectra

Figure 2-15 to Figure 2-19 provide the results (cross-sectional flux on a sphere) of the MASTER model for an ISS-like orbit. All spectra are given for the complete size range of the MASTER model.

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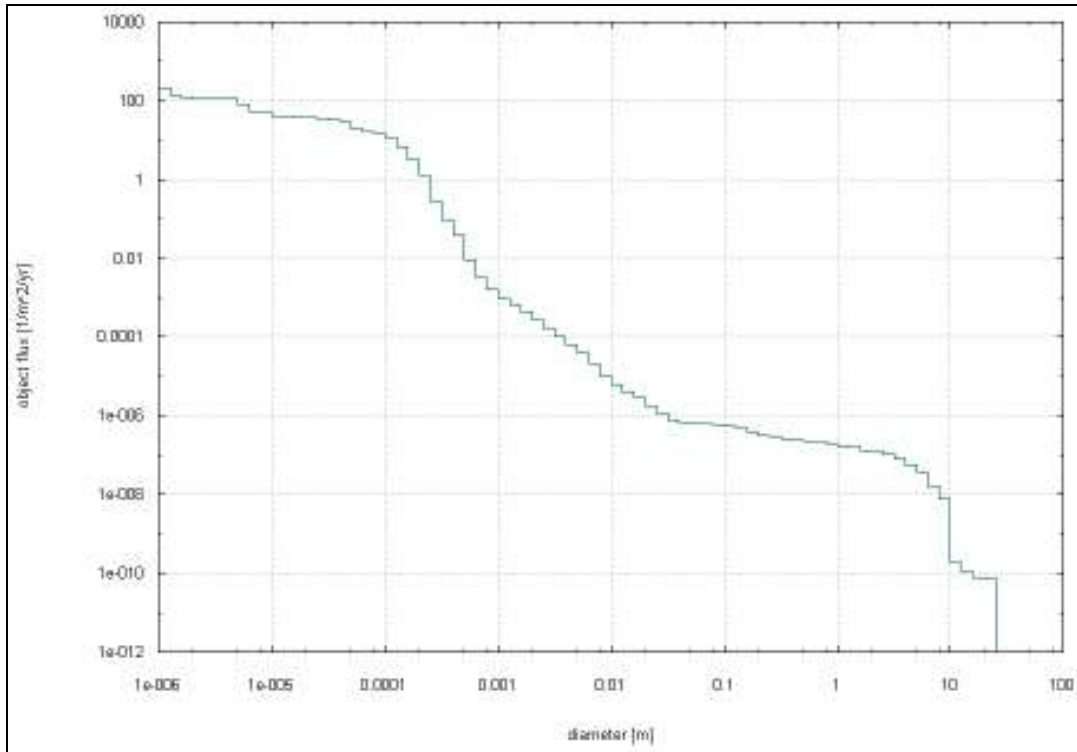


Figure 2-15 MASTER 2005 flux vs. particle diameter, 400 km / 51.6° orbit

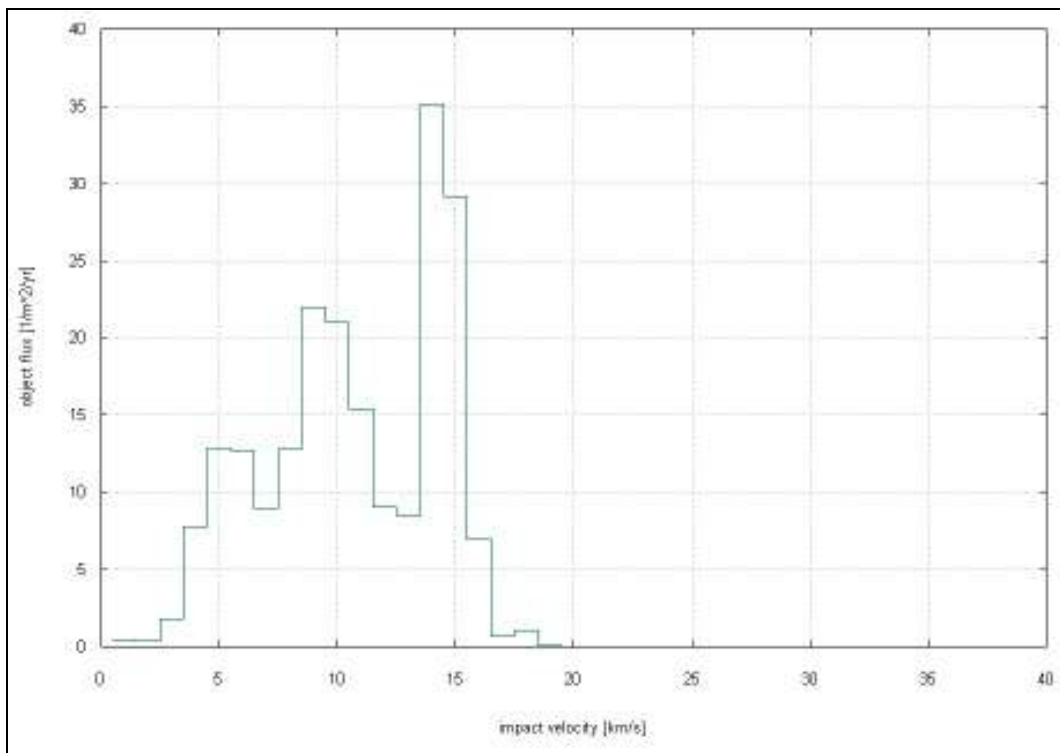


Figure 2-16 MASTER 2005 flux vs. impact velocity, 400 km / 51.6° orbit / d > 1µm

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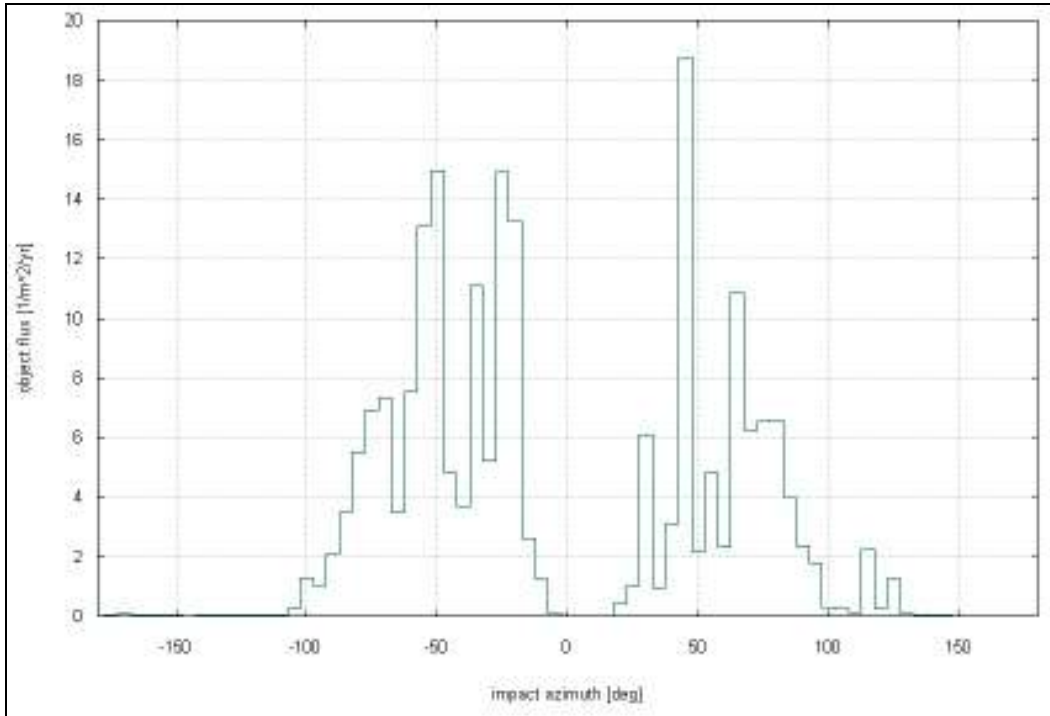


Figure 2-17 MASTER 2005 flux vs. azimuth, 400 km / 51.6° orbit / $d > 1\mu\text{m}$

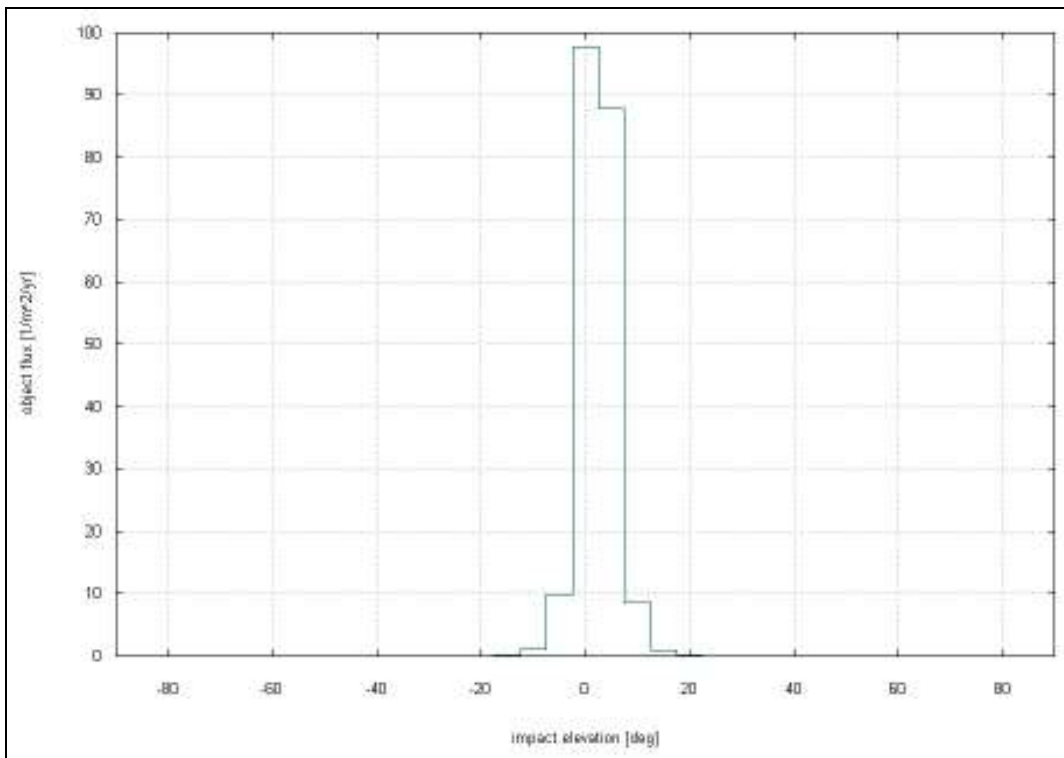


Figure 2-18 MASTER 2005 flux vs. elevation, 400 km / 51.6° orbit / $d > 1\mu\text{m}$

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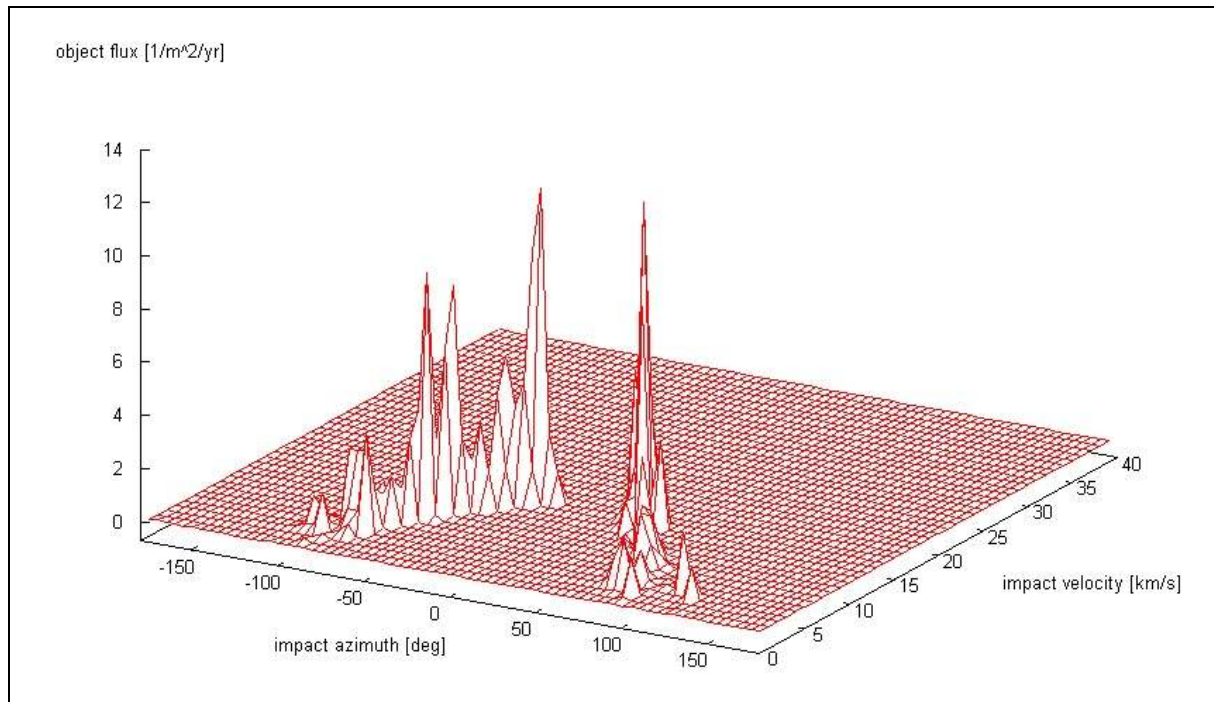


Figure 2-19 MASTER 2005 flux vs. velocity and azimuth, 400 km / 51.6° orbit / $d > 1\mu\text{m}$

Although the spectra are displayed as differential distributions – except the diameter spectrum, which is cumulative – for compatibility with the results of the other debris models (see section 2.1.2.1), the distributions are provided and used in their cumulative form within the ESABASE2 analysis as described in chapter 5.

In ESABASE2/Debris the MASTER 2005 flux analysis is always performed for one complete orbit due to corresponding limitations of the MASTER 2005 flux analysis output. This results in identical analysis results for each orbital point in case of MASTER 2005, while in case of MASTER 2001 and ORDEM 2000 the flux analysis is performed for each orbital point.

2.2 The Meteoroid Environment Models

2.2.1 Introduction

To describe the *sporadic* part of the meteoroid flux the Grün model remains the base. It is briefly recapitulated in section 2.2.2.

Additionally, the Divine-Staubach meteoroid model as provided in the MASTER 2005 application has been implemented into ESABASE/Debris. A description of the main model theory is given in section 2.2.3.

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Furthermore the Meteoroid Engineering Model (MEM) has been implemented into ESABASE/Debris. A brief description is given in section 2.2.4.

For the *meteoroid streams*, which so far have been included in ESABASE by the Cour-Palais method (Ref. /11/) and which did not include directional information, a new approach which is based on more comprehensive and more recent observations evaluated by **P. Jenniskens**, (Ref. /5/) has been worked out by N. McBride (Ref. /6/). The first version of his contribution is dated December 1995. This version has been updated in January 1996 by including some non-symmetrical activity profiles. The latest developments are contained in (Ref. /13/). The description of the streams presented in this document is based on these latter references. For more details, please refer to the original papers or (Ref. /15/).

Some *enhancements* of the *sporadic* contribution with suggestions on how one might include the expected anisotropy in the Earth *apex* direction, the β *meteoroids* that are, because of their small size, driven away from the sun by radiation pressure, and of *interstellar* dust are discussed in a paper by N. McBride and J.A.M. McDonnell (Ref. /9/). The results for these additional directional effects which have been implemented in the software are collected in section 2.2.6 .

Also in Ref. /9/ an enhanced *velocity distribution* is given which was derived from HRMP observation data and was proposed by Taylor in 1995 (Ref. /10/). This distribution is given at 1 AU as seen from a massless Earth. This distribution is added as a new option in the enhanced tool. It is described in section 2.2.7 .

2.2.2 The Grün Model

In 1985 E. Grün et al (Ref. /4/) established an interplanetary flux model which can be considered as today's de facto standard for the modelling of the sporadic meteoroid environment. The model does not give any directional information, it assumes an isotropic environment. However, the annual streams contributions and other directional effects are implicitly contained in the Grün model.

The flux - mass distribution is defined as

$$F(m) = c_0 \{ (c_1 m^{0.306} + c_2)^{4.38} + c_3 (m + c_4 m^2 + c_5 m^4)^{-0.36} + c_6 (m + c_7 m^2)^{-0.85} \} ,$$

$$m > 10^{-9} \text{ g} \quad \rightarrow \quad m > 10^{-14} \text{ g} \quad \rightarrow \quad 10^{-18} \text{ g} < m < 10^{-14} \text{ g} \quad \rightarrow$$

where $F(m)$ is the cumulative flux of particles with mass m or larger in *particles/m²/year* to one side of a *randomly tumbling* plate which is assumed as *stationary* with respect to the Earth surface, m is the mass in g and the constants c_0 to c_7 are defined as follows:

$$c_0 = 3.156 \cdot 10^7 \quad (\text{from conversion of the units from } m^{-2} s^{-1} \text{ to } m^{-2} \text{ year}^{-1})$$

$$c_1 = 2.2 \cdot 10^3$$

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$$\begin{aligned}
 c_2 &= 15 \\
 c_3 &= 1.3 \cdot 10^{-9} \\
 c_4 &= 10^{11} \\
 c_5 &= 10^{27} \\
 c_6 &= 1.3 \cdot 10^{-16} \\
 c_7 &= 10^6
 \end{aligned}$$

In the line below the equation for $F(m)$ the mass contributions of the three terms are indicated. It is seen that the model covers a large mass interval from 10^{-18} to 1 g.

The Grün model represents the total meteoroid influx at the Earth's position i.e. at 1 AU distance from the sun in the ecliptic plane, but in the absence of the Earth. This requires that the flux must be corrected by the focusing effect of the gravitational field of the Earth as well as by the shielding effect of the Earth and its atmosphere. The corresponding equations are given in section 2.2.9 .

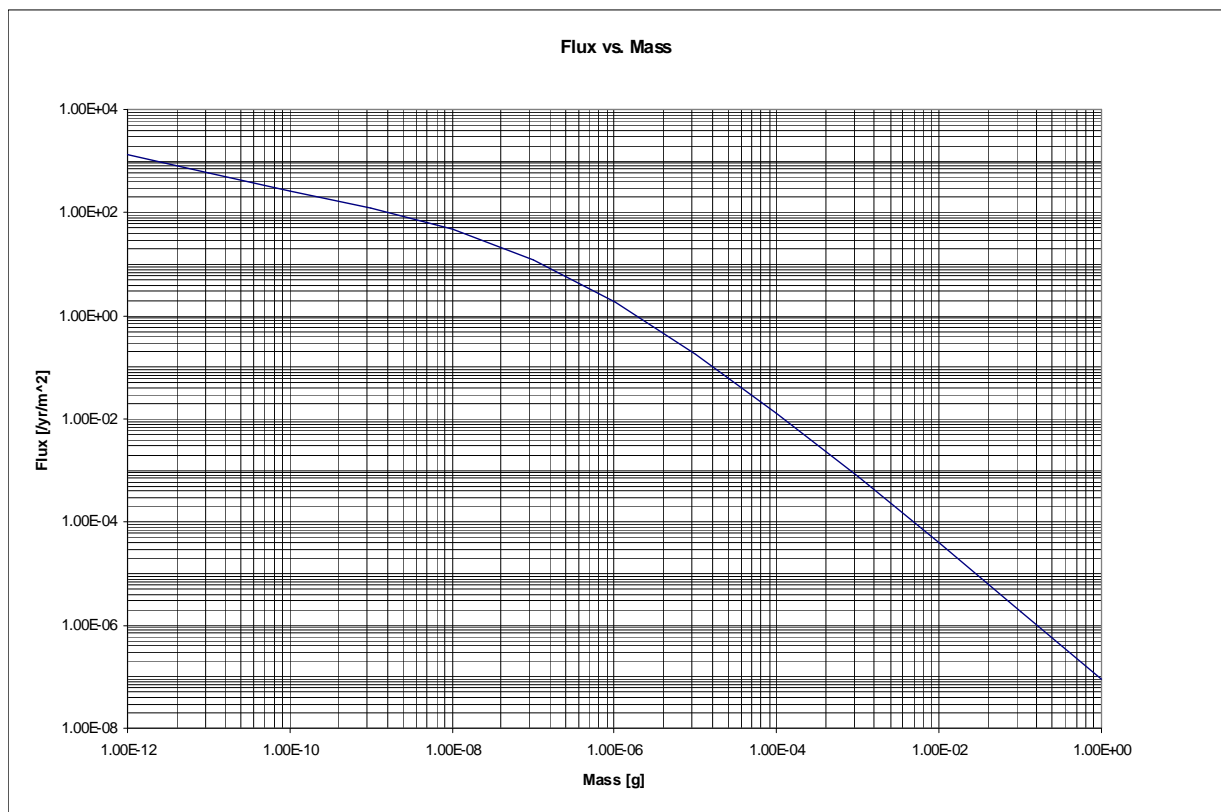


Figure 2-20 Grün flux vs. particle mass, 400 km / 51.6° orbit

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2.2.3 The Divine-Staubach Model

2.2.3.1 Implementation

As mentioned above (section 2.1.3.1), the mathematical approach developed by N. Divine originally has been applied to the interplanetary meteoroid environment. For debris flux calculation with the MASTER 2005 application the approach had to be adapted as described in section 2.1.3.2. The population as well as the target orbit is described in a geo-centric equatorial co-ordinate system. However, the meteoroid population is given in the helio-centric ecliptical co-ordinate system. Thus, the transformation of the target orbit from the geo-centric to the helio-centric system has been implemented for the evaluation of meteoroid flux. (This change, however, is not visible in the above given equations.)

The meteoroid population used by Divine-Staubach is divided into five sub-populations:

- asteroidal population,
- core population,
- A, B, and C population.

The population data is read from an input file as given in /22/. Figure 2-21 gives the mass distribution and the orbital element distributions of the five meteoroid populations of the Divine-Staubach model.

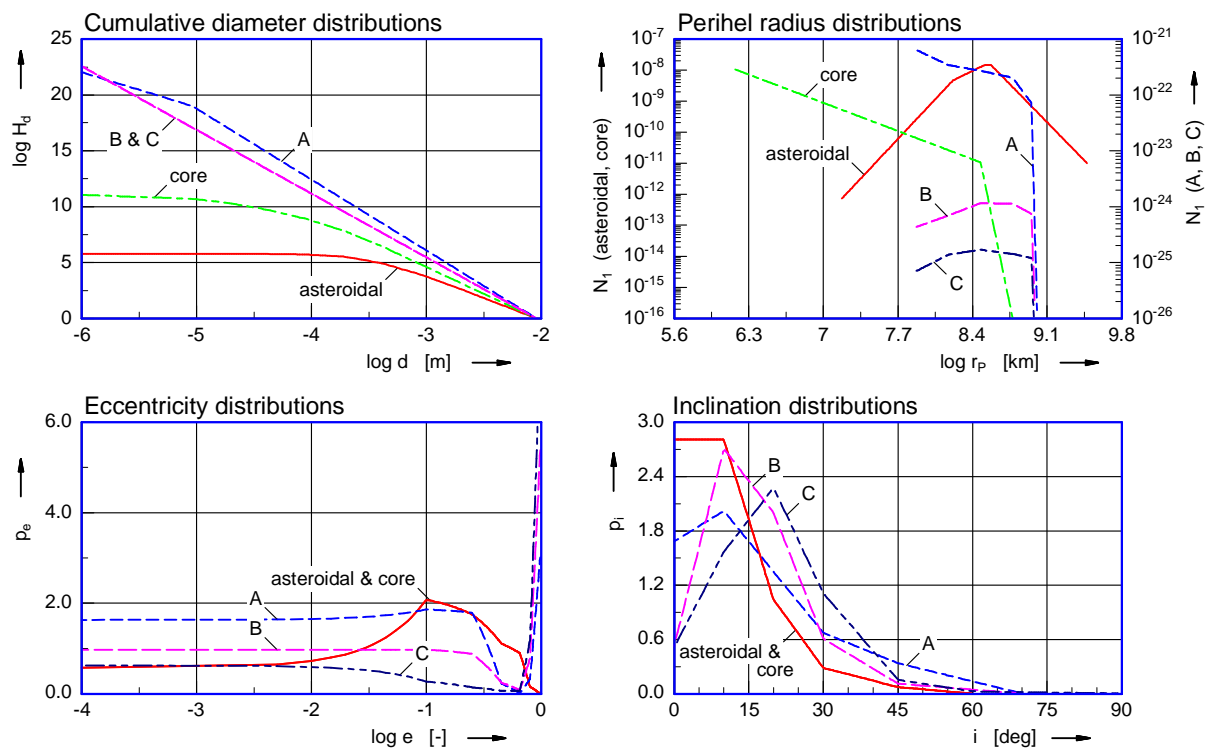


Figure 2-21 The mass and orbital element distributions of the Divine-Staubach model

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The extensions of the theory concerning Earth focussing and shielding as described in /22/ is implemented in the meteoroid branch of the Standard application. The flux equation (1) then becomes for meteoroids

$$J_M = \frac{1}{4} \sum_{dir=1}^4 [N_M \cdot (v_{imp})_{dir} \cdot \eta_F \eta_S] \quad (5)$$

where η_F is the focussing factor

$$\eta_F = \left| 0,5 - \left(v_{FhE} - v_{rFhE} + \frac{2\mu_E}{r_{OhE} v_{FhE}} \right) \left(\frac{\pm 1}{4B} \right) \right| \quad (6)$$

where

$$B = + \sqrt{\left(\frac{v_{FhE} - v_{rFhE}}{4} \right) \left(v_{FhE} - v_{rFhE} + \frac{4\mu_E}{r_{OhE} v_{FhE}} \right)} \quad (7)$$

with

- μ_E gravitational parameter of the Earth ($3,986 \cdot 10^{-5} \text{ km}^3/\text{s}^2$)
- v_{FhE} heliocentric meteoroid velocity with respect to the Earth
- v_{rFhE} radial component of v_{FhE}
- r_{OhE} heliocentric object distance with respect to the Earth

The shielding factor η_S is given by

$$\eta_S = \begin{cases} 0 & \text{if } f_F > 0 \text{ and } r_{pF} < (R_E + H_A) \\ 1 & \text{in all other cases} \end{cases} \quad (8)$$

with

- f_F meteoroid true anomaly
- r_{pF} meteoroid orbit perigee radius
- R_E mean Earth radius (6378,144 km)
- H_A height of the dense Earth atmosphere ($\approx 120 \text{ km}$)

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With these steps, the complete Divine-Staubach model is made available in the MASTER 2001 Standard application.

2.2.3.2 Results

As for the debris population, the MASTER model provides flux results including the directional information by means of the Divine-Staubach model. Figure 2-22 to Figure 2-26 provide the resulting spectra (cross-sectional flux on a sphere) for an ISS-like orbit and a lower particle size threshold of 1 μ m. Please note that the meteoroid flux results are depending on the analysis epoch and the orientation of the target orbit with respect to the ecliptic plane.

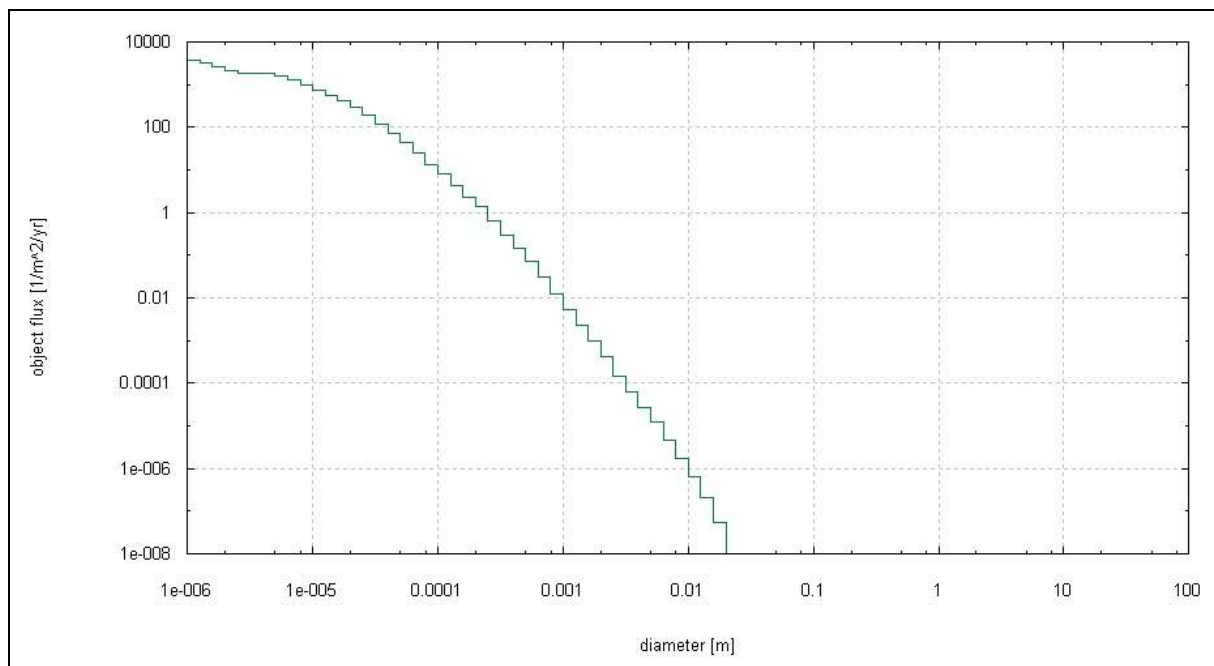


Figure 2-22 Divine-Staubach flux vs. particle diameter, 400 km / 51.6° orbit

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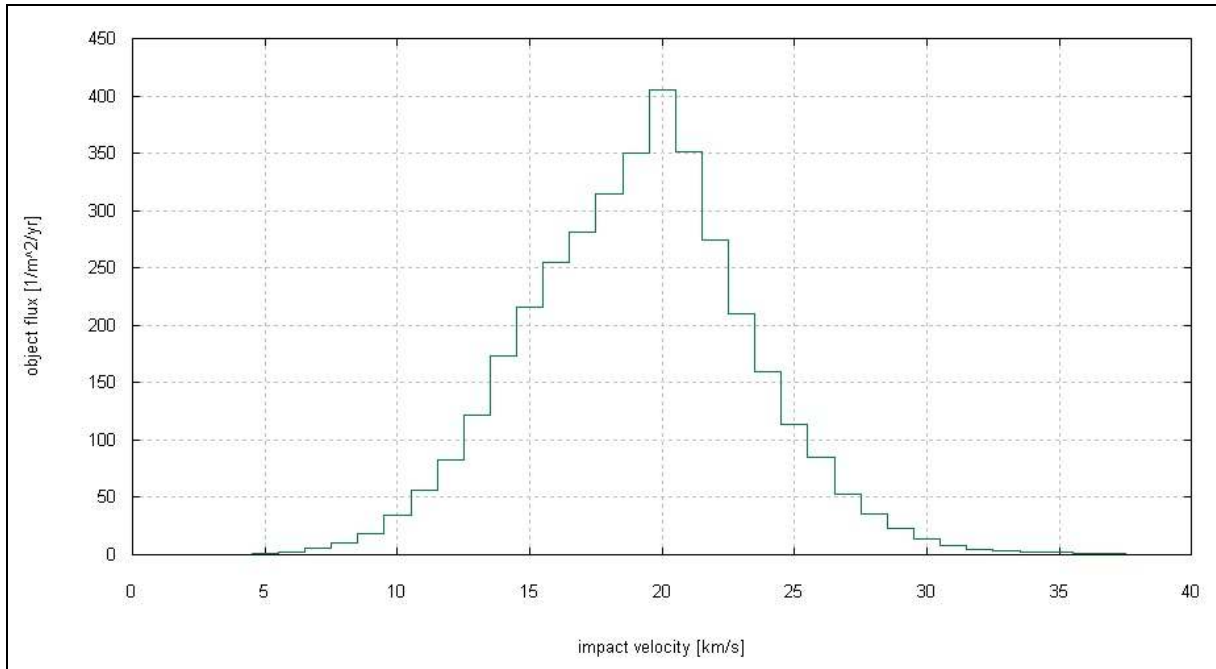


Figure 2-23 Divine-Staubach flux vs. impact velocity, 400 km / 51.6° orbit / $d > 1\mu\text{m}$

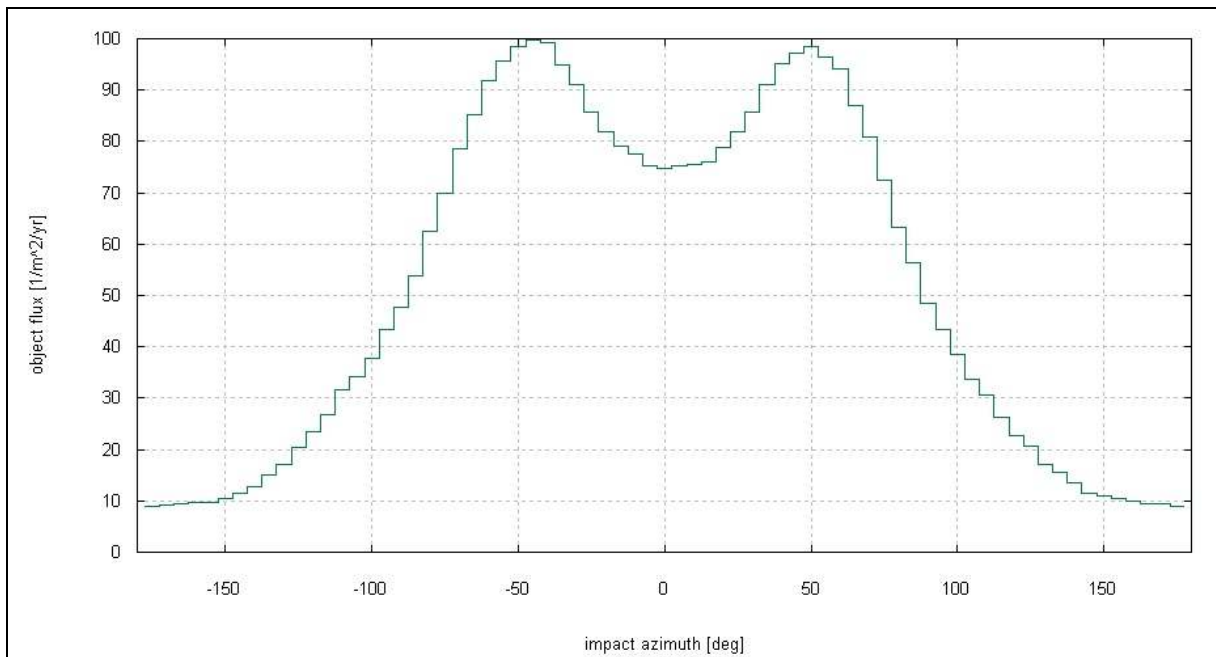


Figure 2-24 Divine-Staubach flux vs. azimuth, 400 km / 51.6° orbit / $d > 1\mu\text{m}$

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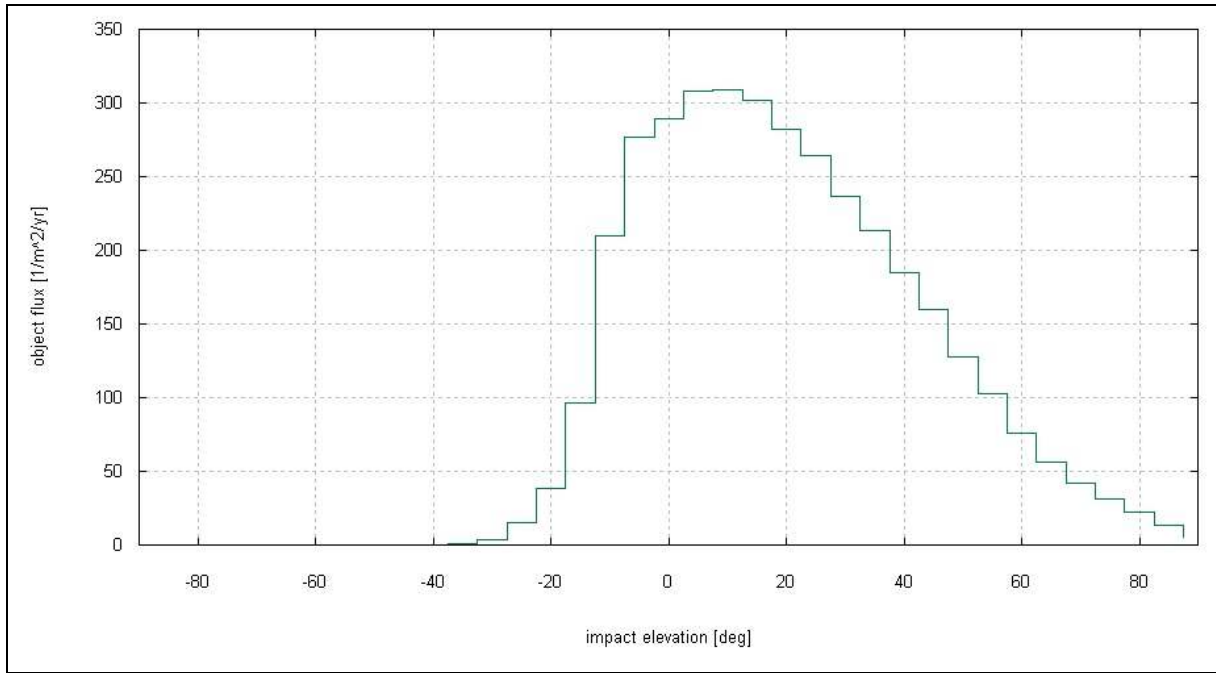


Figure 2-25 Divine-Staubach flux vs. elevation, 400 km / 51.6° orbit / $d > 1\mu\text{m}$

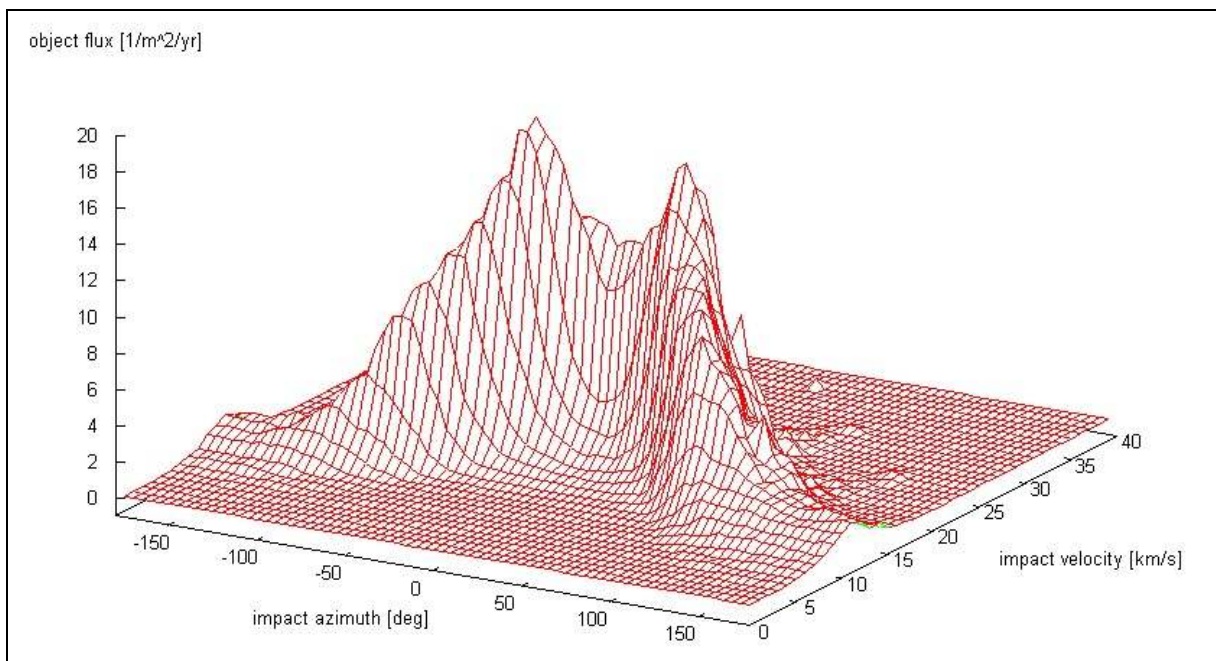


Figure 2-26 Divine-Staubach flux vs. impact velocity and azimuth, ISS orbit / $d > 1\mu\text{m}$

Although the spectra are displayed as differential distributions – except the diameter spectrum, which is cumulative – the distributions are provided and used in their cumulative form within the ESABASE/Debris analysis as described in chapter 5.

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2.2.4 The Meteoroid Model MEM

2.2.4.1 Implementation

MEM incorporates a physics-based approach to modelling the sporadic environment, with validation against radar observations. It predicts the concentration and velocity distribution of meteoroids within the inner solar system from 0.2 to 2.0 AU, using observational measurements to constrain the physical model.

The fundamental core of the program calculates integral meteoroid fluxes and impacting speeds relative to the spacecraft. In this core, meteoroid velocities and spatial densities are derived from distributions of cometary and asteroidal meteoroid orbits. From these relative velocities and spatial densities, a meteoroid flux is calculated at the spacecraft location. This calculated meteoroid flux (including the directional information) is used as input for ESABASE2/Debris analyses. The model is capable of computing the flux of mass ranges damaging to spacecraft, 10^{-6} g to 10 g. Further details can be found in /36/ and/37/.

MEM is available as executable and the data transfer (input / output) is managed via files. This approach is similar to the one used for ORDEM 2000, MASTER 2005 and Divine-Staubach.

2.2.4.2 Results

MEM provides flux results as multidimensional output. For each elevation/azimuth grid point a complete velocity distribution (flux vs. velocity) is provided. With this input all dependencies between elevation, azimuth and velocity can be considered. The used raytracing procedure is described in chapter 5. The following figures are based on an ISS like orbit.

MEM uses the same normalized flux vs. mass distribution for every orbital point. Figure 2-27 shows the normalized flux vs. mass distribution. Figure 2-28 shows the 2D flux vs. elevation spectrum generated from the multidimensional spectrum. Figure 2-29 shows the cumulated 3D flux vs. elevation/azimuth spectrum as well generated from the multidimensional spectrum. Figure 2-30 shows the flux vs. velocity as well extracted from the multidimensional spectrum.

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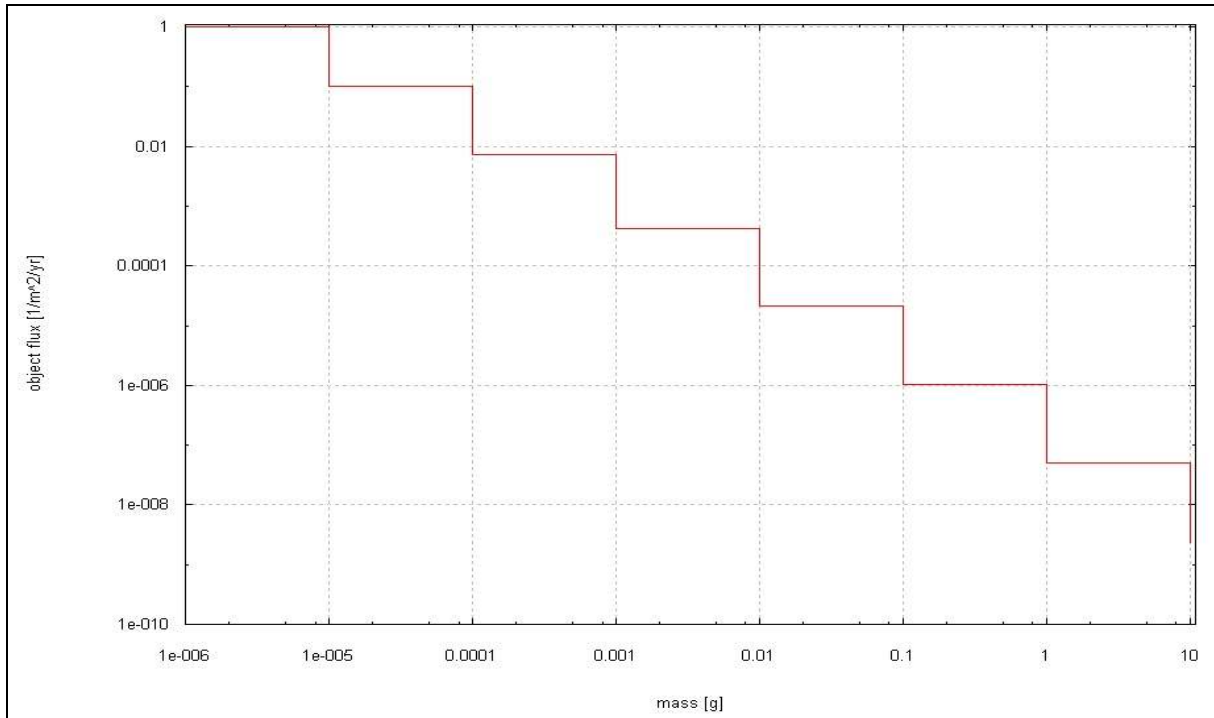


Figure 2-27: MEM normalized flux vs. mass, ISS orbit / mass > 1*10-g

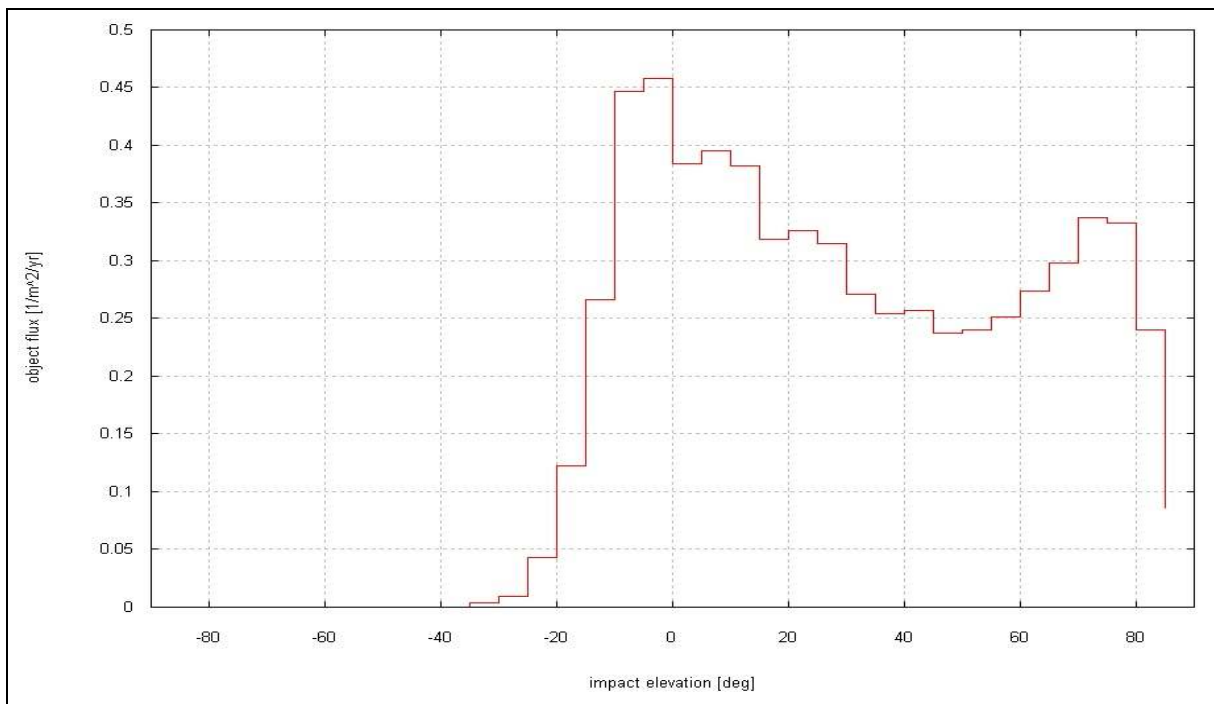


Figure 2-28: MEM flux vs. impact elevation, ISS orbit / mass > 1*10-g

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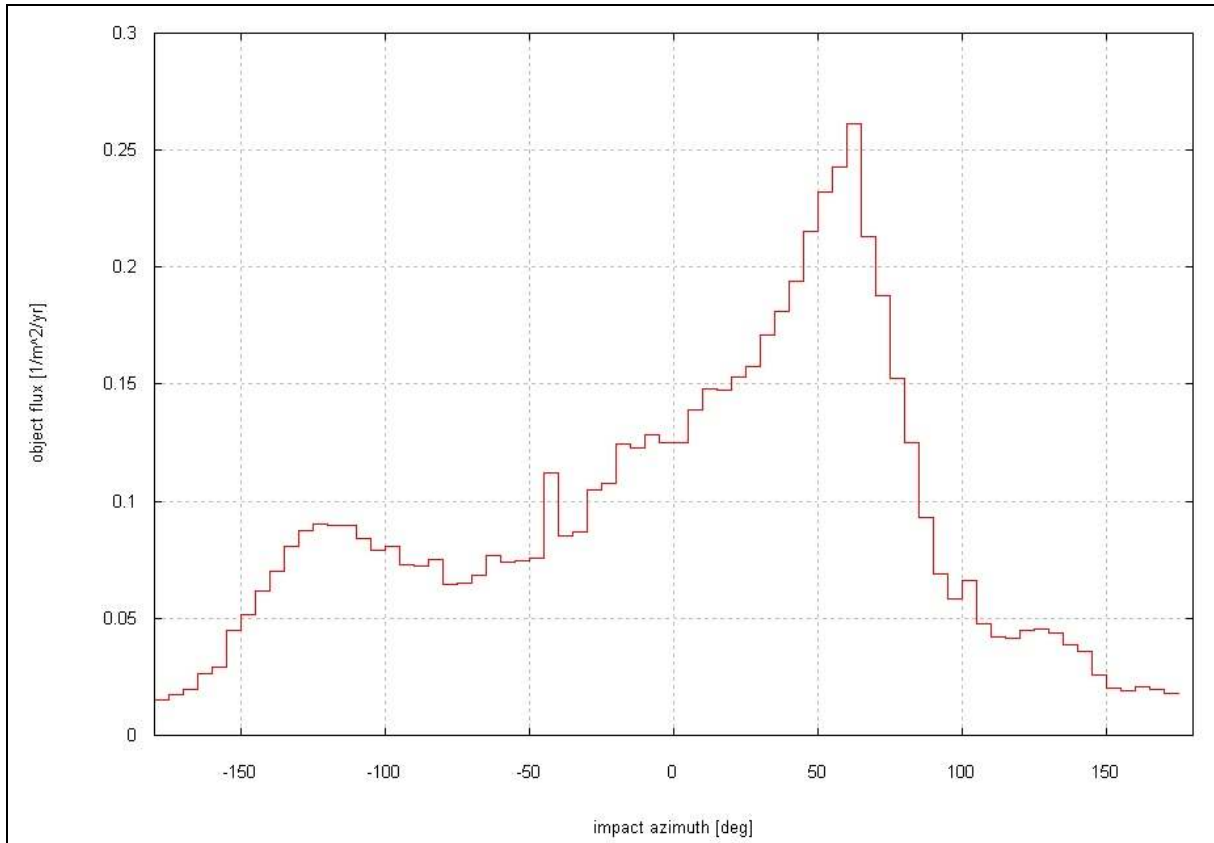


Figure 2-29: MEM cumulated flux vs. impact azimuth, ISS orbit / mass >1*10-g

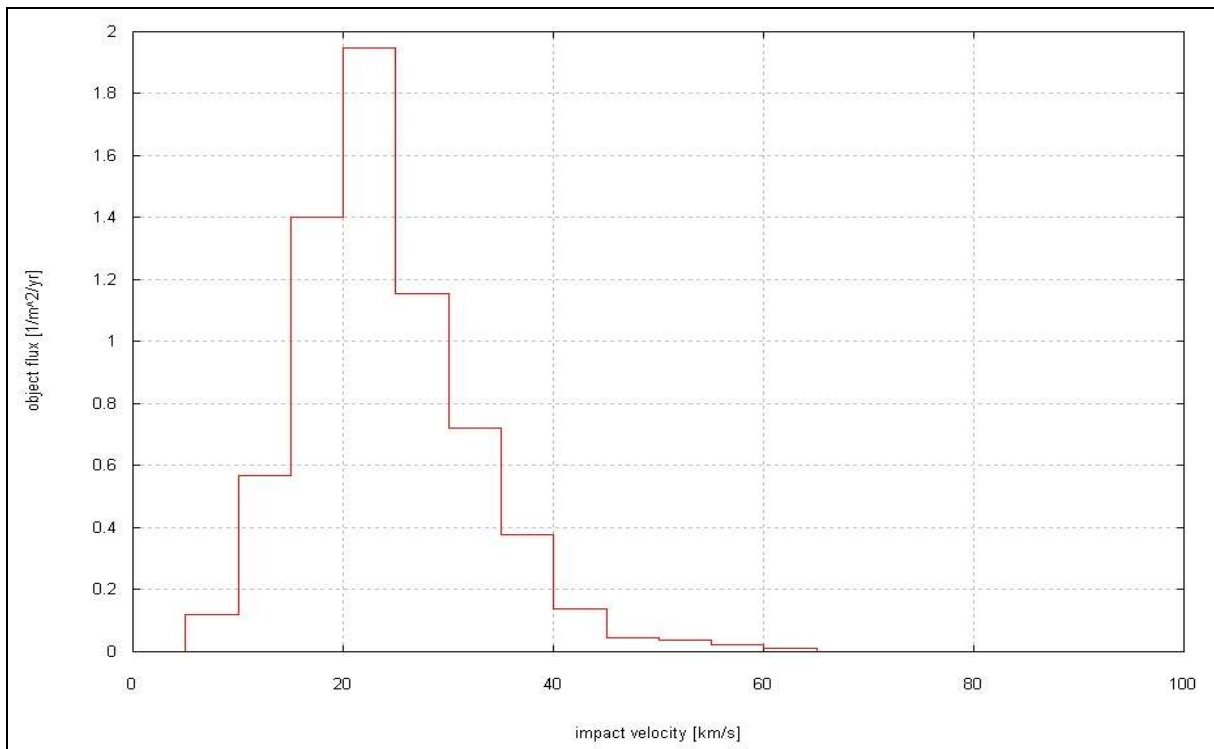


Figure 2-30: MEM flux vs. impact velocity, ISS orbit / mass >1*10-g

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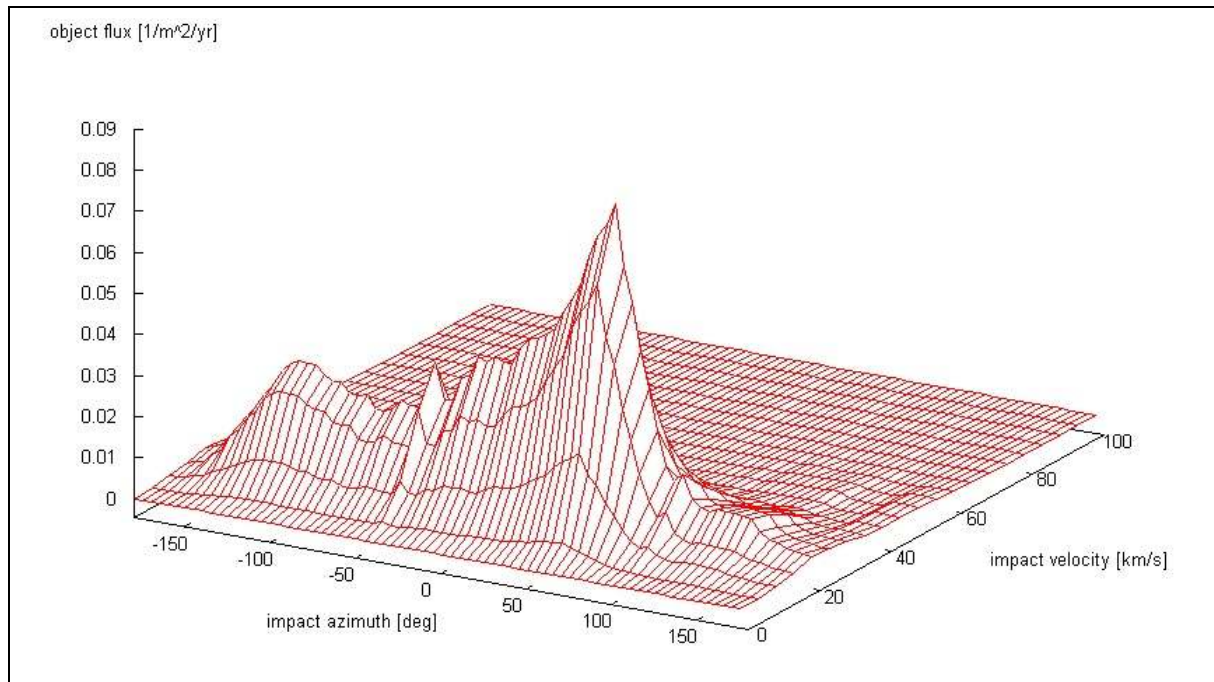


Figure 2-31: MEM flux vs. impact azimuth and velocity, ISS orbit / mass >1*10-g

Although the spectra are displayed as differential distributions – except the diameter spectrum, which is cumulative – the distributions are provided and used in their cumulative form within the ESABASE/Debris analysis as described in chapter 5

2.2.5 Meteoroid Streams According to Jenniskens/McBride

2.2.5.1 General

In the old ESABASE/Debris software the annual meteoroid streams are implemented according to the Cour-Palais 1969 method (Ref. /11/) which does not include directional effects. This has been replaced by a new approach of P. Jenniskens (Ref. /5/, 1994) which is based on data collected by a large number of observers over a 10 year period from observation sites in both the northern and southern hemispheres. In Ref. /13/ N. McBride describes how the parameters of Jenniskens have to be implemented into a numerical application.

In summary the stream geometry and activity at shower maximum is defined by:

- a) the solar longitude λ at shower maximum λ_{\max}
- b) the maximum zenithal hourly rate ZHR_{\max} , which is the number of 'visible' meteors seen after various observer and location related corrections have been applied
- c) apparent radiant position in RA (right ascension of the radiant) and Dec (declination of the radiant). These values are tabulated in Table 6 at an epoch defined by the solar longitude λ^0

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d) the geocentric meteoroid speeds, defined as the final geocentric velocity V_{∞} as the meteoroids reach the top of the atmosphere

The right ascension of the radiant and for the declination for an instantaneous value of the solar longitude λ are obtained by

$$RA(\lambda) = RA(\lambda^0) + \Delta RA (\lambda - \lambda^0), \quad Dec(\lambda) = Dec(\lambda^0) + \Delta Dec (\lambda - \lambda^0).$$

The shower activity as a function of time around its maximum is described by

$$ZHR = ZHR_{\max} 10^{-B|\lambda - \lambda_{\max}|},$$

where B is given in Ref. /5/ and describes the slopes of the activity profiles. Since most streams are found to have symmetrical profiles a single value of B is sufficient. The Geminids are the exception; this stream needs a different value of B for the inward and outward slope. Six of the streams do not have a strong enough ZHR to produce a slope, here it is suggested to use a 'typical' value of $B = 0.2$. Six other streams are best represented by the sum of 2 activity profiles, defined by a peak profile ZHR_{\max}^p and B^p and a background profile ZHR_{\max}^b with separate inward and outward slope values B^{b+} and B^{b-} respectively. This results in the following expression:

$$ZHR = ZHR_{\max}^p 10^{-B^p|\lambda - \lambda_{\max}|} + ZHR_{\max}^b (10^{-B^{b-}(\lambda_{\max} - \lambda)} + 10^{-B^{b+}(\lambda - \lambda_{\max})})$$

The cumulative flux at solar longitude λ can now be expressed as

$$R(m, \lambda) = R(m)_{\max} \frac{ZHR(\lambda)}{ZHR_{\max}}$$

with

$$R(m)_{\max} = k m^{-\alpha}$$

The total particle flux F_{TOT} is obtained by summation over all streams

$$F_{TOT} = F_{SPORADIC} + \sum F_{ST}.$$

Since the Grün flux models all particles, including the streams, F_{TOT} must be forced to equal the Grün flux when summed over a full year. Thus, when the stream model is used, the new sporadic flux becomes

$$F_{SPORADIC} = F_{Grün} - \sum_{1\text{year}} F_{ST}$$

where the sum is to be evaluated over one full year.

Note: Additional streams may be defined in place of or in addition to the Jenniskens streams by the user, using the format of Table 6, see ref. /14/.

λ_{\max} RA_{\max} ΔRA Dec_{\max} ΔDec ZHR_{\max}^p B^{p+} B^{p-} ZHR_{\max}^b B^{b+} B^{b-} α k V_{∞}

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Bootids	283.3	232.	0.6	45.	-0.31	10.0	2.50	2.50	20.0	.37	.45	.92	.84·10 ⁻¹⁶	43
γVelids	285.7	124.	0.5	-47.	-0.2	2.4	.12	.12	0.0	.0	.0	1.10	.58·10 ⁻¹⁸	35
αCrucids	294.5	193.	1.1	-63.	-0.4	3.0	.11	.11	0.0	.0	.0	1.06	.19·10 ⁻¹⁸	50
αHydrusids	300.0	138.	0.7	-13.	-0.3	2.0	.20	.20	0.0	.0	.0	1.03	.34·10 ⁻¹⁸	44
αCarinids	311.2	99.	0.4	-54.	0.0	2.3	.16	.16	0.0	.0	.0	.92	.13·10 ⁻¹⁶	25
δVelids	318.0	127.	0.5	-50.	-0.3	1.3	.20	.20	0.0	.0	.0	1.10	.31·10 ⁻¹⁸	35
αCentaurids	319.4	210.	1.3	-58.	-0.3	7.3	.18	.18	0.0	.0	.0	.83	.37·10 ⁻¹⁷	57
OCentaurids	323.4	176.	0.9	-55.	-0.4	2.2	.15	.15	0.0	.0	.0	1.03	.19·10 ⁻¹⁸	51
θCentaurids	334.0	220.	1.1	-44.	-0.4	4.5	.20	.20	0.0	.0	.0	.95	.44·10 ⁻¹⁸	60
δLeonids	335.0	169.	1.0	17.	-0.3	1.1	.049	.049	0.0	.0	.0	1.10	.19·10 ⁻¹⁷	23
Virginids	340.0	165.	0.9	9.	-0.2	1.5	.20	.20	0.0	.0	.0	1.10	.15·10 ⁻¹⁷	26
γNormids	353.0	285.	1.3	-56.	-0.2	5.8	.19	.19	0.0	.0	.0	.87	.19·10 ⁻¹⁷	56
δPavonids	11.1	311.	1.6	-63.	-0.2	5.3	.075	.075	0.0	.0	.0	.95	.51·10 ⁻¹⁸	60
Lyrids	32.4	274.	1.2	33.	0.2	12.8	.22	.22	0.0	.0	.0	.99	.20·10 ⁻¹⁷	49
μVirginids	40.0	230.	0.5	-8.	-0.3	2.2	.045	.045	0.0	.0	.0	1.10	.11·10 ⁻¹⁷	30
ηAquarids	46.5	340.	0.9	-1.	0.3	36.7	.08	.08	0.0	.0	.0	.99	.15·10 ⁻¹⁷	66
βCorona Aust.	56.0	284.	1.3	-40.	0.1	3.0	.20	.20	0.0	.0	.0	1.13	.15·10 ⁻¹⁸	45
αScorpiids	55.9	252.	1.1	-27.	-0.2	3.2	.13	.13	0.0	.0	.0	.92	.47·10 ⁻¹⁶	21
Da.Arietids	77.0	47.	0.7	24.	0.6	54.0	.10	.10	0.0	.0	.0	.99	.26·10 ⁻¹⁶	38
γSagitarids	89.2	286.	1.1	-25.	0.1	2.4	.037	.037	0.0	.0	.0	1.06	.19·10 ⁻¹⁷	29
τCetids	95.7	24.	0.9	-12.	0.4	3.6	.18	.18	0.0	.0	.0	.92	.37·10 ⁻¹⁸	66
θOphiuchids	98.0	292.	1.1	-11.	0.1	2.3	.037	.037	0.0	.0	.0	1.03	.35·10 ⁻¹⁷	27
τAquarids	98.0	342.	1.0	-12.	0.4	7.1	.24	.24	0.0	.0	.0	.92	.89·10 ⁻¹⁸	63
νPhoenicids	111.2	28.	1.0	-40.	0.5	5.0	.25	.25	0.0	.0	.0	1.10	.26·10 ⁻¹⁸	48
OCygnids	116.7	305.	0.6	47.	0.2	2.5	.13	.13	0.0	.0	.0	.99	.14·10 ⁻¹⁷	37
Capricornid	122.4	302.	0.9	-10.	0.3	2.2	.041	.041	0.0	.0	.0	.69	.83·10 ⁻¹⁶	25
τAquarids N	124.1	324.	1.0	-8.	0.2	1.0	.063	.063	0.0	.0	.0	1.19	.36·10 ⁻¹⁹	42
Pisces Aust.	124.4	339.	1.0	-33.	0.4	2.0	.40	.40	0.9	.03	.10	1.16	.15·10 ⁻¹⁸	42
δAquarids S.	125.6	340.	0.8	-17.	0.2	11.4	.091	.091	0.0	.0	.0	1.19	.36·10 ⁻¹⁸	43
ιAquarids S.	131.7	335.	1.0	-15.	0.3	1.5	.07	.07	0.0	.0	.0	1.19	.12·10 ⁻¹⁸	36
Perseids	140.2	47.	1.3	58.	0.1	70.0	.35	.35	23.0	.05	.092	.92	.12·10 ⁻¹⁶	61
κCygnids	146.7	290.	0.6	52.	0.3	2.3	.069	.069	0.0	.0	.0	.79	.30·10 ⁻¹⁶	27
πEridanids	153.0	51.	0.8	-16.	0.3	40.0	.20	.20	0.0	.0	.0	1.03	.17·10 ⁻¹⁷	59
γDoradids	155.7	60.	0.5	-50.	0.2	4.8	.18	.18	0.0	.0	.0	1.03	.11·10 ⁻¹⁷	41
Aurigids	158.2	73.	1.0	43.	0.2	9.0	.19	.19	0.0	.0	.0	.99	.29·10 ⁻¹⁸	69
κAquarids	177.2	339.	0.9	-5.	0.4	2.7	.11	.11	0.0	.0	.0	1.03	.19·10 ⁻¹⁶	19
εGeminids	206.7	104.	0.7	28.	0.1	2.9	.082	.082	0.0	.0	.0	1.10	.21·10 ⁻¹⁹	71
Orionids	208.6	96.	0.7	16.	0.1	25.0	.12	.12	0.0	.0	.0	1.13	.16·10 ⁻¹⁸	67
Leo Minorids	209.7	161.	1.0	38.	-0.4	1.9	.14	.14	0.0	.0	.0	.99	.11·10 ⁻¹⁸	61
Taurids	223.6	50.	0.3	18.	0.1	7.3	.026	.026	0.0	.0	.0	.83	.43·10 ⁻¹⁶	30
δEridanids	229.0	54.	0.9	-2.	0.2	0.9	.20	.20	0.0	.0	.0	1.03	.75·10 ⁻¹⁸	31
ζPuppids	232.2	117.	0.7	-42.	-0.2	3.2	.13	.13	0.0	.0	.0	1.22	.95·10 ⁻¹⁹	41
Leonids	235.1	154.	1.0	22.	0.4	19.0	.55	.55	4.0	.025	.15	1.22	.34·10 ⁻¹⁹	71
Puppids/Vel	252.0	128.	0.8	-42.	-0.4	4.5	.034	.034	0.0	.0	.0	1.06	.82·10 ⁻¹⁸	40
Phoenicids	252.4	19.	0.8	-58.	0.4	2.8	.30	.30	0.0	.0	.0	1.03	.25·10 ⁻¹⁶	18
Monocerotid.	260.9	100.	1.0	14.	-0.1	2.0	.25	.25	0.0	.0	.0	1.25	.33·10 ⁻¹⁹	43
Geminids	262.1	113.	1.0	32.	0.1	74.0	.59	.81	18.0	.09	.31	.95	.78·10 ⁻¹⁶	36
σHydrusids	265.5	133.	0.9	0.	-0.3	2.5	.10	.10	0.0	.0	.0	1.10	.47·10 ⁻¹⁹	59
Ursids	271.0	224.	-0.2	78.	-0.3	10.0	.90	.90	2.0	.08	.2	1.22	.81·10 ⁻¹⁸	35

Table 6 The 50 Jenniskens Streams

2.2.5.2 Implementation

The following symbols are used in Table 6 and in the formulas that follow:

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λ	solar longitude in degrees
λ_{\max}	solar longitude in degrees at shower peak (tabled in Table 6 for 50 streams)
$ZHR_{\max}^{p/b}$	Zenith Hourly Rate at shower peak / background (observer corrected) <u>Note:</u> the b (background) index is only relevant for streams having two profiles.
RA	right ascension of the radiant at solar longitude λ in degrees
Dec	declination of the radiant at solar longitude λ in degrees
ΔRA	variation of RA per degree of solar longitude λ
ΔDec	variation of Dec per degree of solar longitude λ
$B, B^p, B^{p+}, B^{p-}, B^{b+}, B^{b-}$	slopes of the shower activity profiles
V_{∞}	meteoroid arrival velocity in km/s, already containing gravitational enhancement
$F(m)$	cumulative flux in $m^{-2} s^{-1}$ of particles with mass greater than m (kg)
α	cumulative mass distribution index
k	cumulative mass distribution constant
$Q = ZHR / ZHR_{\max}^i$	Ratio of actual ZHR to its peak value; index $i = p$ or b

The following algorithm now applies to determine the individual streams' fluxes:

- 1) Given λ , choose the closest value of λ_{\max} in the Table and determine the stream number
- 2) From $\Delta\lambda = 2/B$ determine if λ is within the range

$$(\lambda_{\max} - \Delta\lambda) < \lambda < (\lambda_{\max} + \Delta\lambda) \quad (\Delta\lambda \text{ determined by } 1\% \text{ of } ZHR_{\max})$$

if not, skip this stream. (λ_{\max} to be taken from Table 6)

- 3) Calculate ZHR within the profile

$$ZHR = ZHR_{\max}^p 10^{-B|\lambda - \lambda_{\max}|}$$

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3a) For the six streams in Table 6 which have two activity profiles (non vanishing B^{b+} / B^{b-} values), calculate according to equation 1 and equation 10 the ratio

$$Q = ZHR / ZHR_{\max}^p \cdot 10^{-B^p |\lambda - \lambda_{\max}|} + ZHR / ZHR_{\max}^b (10^{-B^{b-} (\lambda_{\max} - \lambda)} + 10^{-B^{b+} (\lambda - \lambda_{\max})})$$

4) The cumulative flux is now given by

$$F(m) = F(m)_{\max} \cdot Q$$

with

$$F(m)_{\max} = k m^{-\alpha} \quad \text{or} \quad dF = -\alpha k m^{-(\alpha+1)} dm$$

k and α are obtained from the Table according to the relevant stream number.

The *arrival velocity* V_{∞} does not need gravitational corrections due to the Earth gravity because it is measured at the top of the atmosphere.

2.2.6 Further Directional Effects

2.2.6.1 Introduction

From Ref. /9/ it becomes apparent that actually very little is known about how the Grün flux should be modified to include an apex enhancement, to sort out the beta meteoroids, and to include interstellar dust.

2.2.6.2 Separation of α - and β -Source

In Ref. /9/ it is suggested to separate the Grün flux into an α population and an β population which has a crossover at 10^{-11} g. The β population has the direction from the Sun and is of the small particle size. The separation into the α - flux $F_{\alpha}(M)$ and the β - flux $F_{\beta}(M)$ may then be done in the following way (Ref. /9/ , eqs, 25,26,27):

$$F_{\beta}(M) = F_G(M) - F_{\alpha}(M) \quad F_G(M) = \text{Grün flux}$$

$$F_{\alpha}(M) = \frac{F_G(M) \cdot F_H(M)}{F_G(M) + F_H(M)}$$

$$\log(F_H) = -a \log(M) - b \quad a = 0.146 \quad b = 6.427$$

Where the units are $F: m^{-2} s^{-1}$, $M: g$

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From the above equations it is possible for each randomly generated mass value to calculate a corresponding cumulative flux value F_α and a corresponding flux value F_β from the Grün flux F_G .

The velocity of the β particles is size dependent, and according to Ref. /9/, Eq. 28 one may assume

$$v(M) = v_0 \left(\frac{M}{10^{-11}} \right)^{-\gamma} \quad v_0 = 20 \text{ km/s}, \quad \gamma = 0.18$$

where v_0 and γ are user supplied.

2.2.6.3 Apex Enhancement of the α -Source

A minimum to maximum antapex to apex flux ratio R_F , which in fact is not known, is used in Ref. /9/ Eqs. 30 - 35 to define a modulation of the flux and of the velocity about the apex direction. The angular deviation from the apex direction is denoted by t , and it is assumed that a parameter δ may describe a slight deviation from the measured peak value which was observed to be about 10° off the apex direction. Thus the modulation of the α flux and of the velocity may be defined as follows:

$$F_\alpha(t) = F_\alpha^0 [1 + \Delta_t \cos(t + \delta)]$$

$$V_\alpha(t) = V_\alpha^0 [1 + \Delta_v \cos(t + \delta)]$$

where $\Delta_t = \frac{R_F - 1}{R_F + 1}$ and $\Delta_v = \frac{V_A - V_{AA}}{V_A + V_{AA}}$

$$V_0 = (V_A + V_{AA}) / 2$$

(subscript A for apex and AA for antapex)

From the AMOR meteor data there are some guesses for the maximum to minimum detection ratio, from which one may try to obtain some values for R_F and V_{AA} . Although R_F could be anywhere in the range of 1 to 5, and V_A and V_{AA} are not known either, it is recommended, that for a first guess one may use the following values:

$$V_A = 17.7 \text{ Km/s}, \quad V_{AA} = 8.3 \text{ km/s}, \quad R_F = \frac{V_A}{V_{AA}} = 2$$

resulting in the following values for Δ_t and Δ_v :

$$\Delta_t = 0.33 \quad \text{and} \quad \Delta_v = 0.36$$

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2.2.6.4 Interstellar Dust

Two components of interstellar dust particles have been observed according to Ref. /9/.

The *first source* concerns measurements on Ulysses and Galileo, which detected at about 5 AU particles of $3 \cdot 10^{-16}$ g with heliocentric velocities of 26 km/s. The ecliptic longitude was 252° and the latitude 2.5° . For the total particle flux at 1 AU one can estimate $5 \cdot 10^{-4} \text{ m}^{-2} \text{ s}^{-1}$, and the heliocentric velocity at 1 AU would be 47 km/s. The mean particle mass detected by the Ulysses dust detector was $3 \cdot 10^{-16}$ g.

The *second source* (Ref. /12/) stems from AMOR meteor data which indicate at least two sources which would be defined by:

radiant direction: $\lambda = 243^\circ$, $\beta = 50^\circ$, $V_\infty = 40 \text{ km/s}$

and

radiant direction: $\lambda = 347^\circ$, $\beta = 60^\circ$, $V_\infty = 80 \text{ km/s}$

λ and β being the ecliptic longitude and latitude respectively.

In Ref. /12/ the mass of these interstellar meteoroids is estimated to lie between 15 and 40 μm . As of today no flux is known for these contributions.

The implementation of the interstellar source in the ESABASE2/Debris tool is lined out below:

The interstellar contributions are given in the ecliptic system where the velocity, the ecliptic longitude and the ecliptic declination of the interplanetary inward direction are input (by the user, see ref. /14/). By using two-body dynamics these quantities are transformed to the Earth reference frame. Unlike the streams contributions, these showers are changing over time in their direction and velocity due to the moving Earth.

2.2.7 Velocity Distributions

2.2.7.1 Constant Velocity

The ESABASE2/Debris tool includes the option of a constant meteoroid velocity, which is input by the user. The mean velocity of 17 km/s of the NASA 90 distribution (see 2.2.5.2 below) yields good results with the Grün flux model.

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2.2.7.2 NASA 90 Meteoroid Velocity Distribution

This normalised distribution is defined in Ref. /3/ and covers a non-vanishing velocity range from 11.1 to 72.2 km/s with a mean velocity of $v = 17$ km/s .

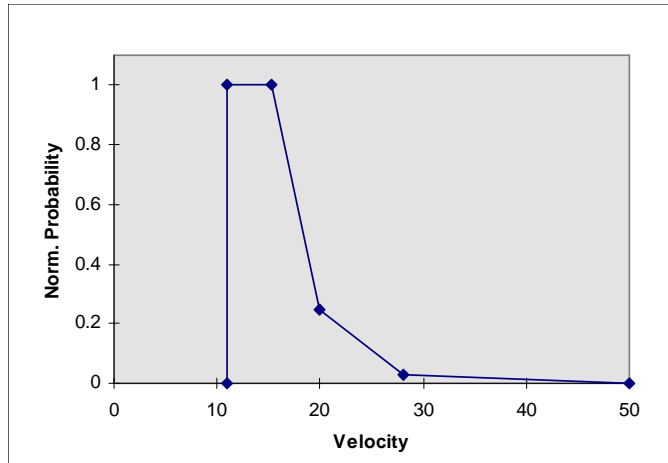


Figure 2-32 NASA 90 velocity distribution

$$g(v) = 0.112 \quad \text{for } 11.1 \leq v < 16.3 \text{ km/s}$$

$$g(v) = 3.328 \cdot 10^5 v^{-5.34} \quad \text{for } 16.3 \leq v < 55 \text{ km/s}$$

$$g(v) = 1.695 \cdot 10^{-4} \quad \text{for } 55 \leq v < 72.2 \text{ km/s}$$

2.2.7.3 Velocity and Flux Distribution According to Taylor

In Ref. /9/ several velocity distributions are discussed. The velocity distribution of meteoroids at 1 AU (i.e. as viewed from a massless Earth) has generally been derived from ground based observations of photographic meteors, which are then corrected for the effect of the Earth gravity. Distributions by Erickson, Sekania and Southworth and by Taylor are compared. It is concluded that the most statistically reliable published data set comes from the Harvard Radio Meteor Project (HRMP) where about 20000 meteor observations were evaluated. Taylor re-evaluated and corrected the original measurement data in Ref./10/ which leads finally to the normalised distribution which is tabulated below at 1 AU, i.e. as seen from a massless Earth. The velocity is in km/s and describes the middle of the 1 km/s wide bin. Each value of $n(v_\infty)$ describes the relative flux of particles within the corresponding bin of 1 km/s width .

v_∞	$n(v_\infty)$	v_∞	$n(v_\infty)$	v_∞	$n(v_\infty)$	v_∞	$n(v_\infty)$
0.5	$0.722 \cdot 10^{-03}$	18.5	$0.447 \cdot 10^{-01}$	36.5	$0.491 \cdot 10^{-02}$	54.5	$0.345 \cdot 10^{-03}$
1.5	$0.227 \cdot 10^{-02}$	19.5	$0.422 \cdot 10^{-01}$	37.5	$0.403 \cdot 10^{-02}$	55.5	$0.326 \cdot 10^{-03}$
2.5	$0.515 \cdot 10^{-02}$	20.5	$0.394 \cdot 10^{-01}$	38.5	$0.330 \cdot 10^{-02}$	56.5	$0.298 \cdot 10^{-03}$
3.5	$0.944 \cdot 10^{-02}$	21.5	$0.363 \cdot 10^{-01}$	39.5	$0.267 \cdot 10^{-02}$	57.5	$0.266 \cdot 10^{-03}$

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4.5	$0.149 \cdot 10^{-01}$	22.5	$0.329 \cdot 10^{-01}$	40.5	$0.214 \cdot 10^{-02}$	58.5	$0.238 \cdot 10^{-03}$
5.5	$0.209 \cdot 10^{-01}$	23.5	$0.297 \cdot 10^{-01}$	41.5	$0.168 \cdot 10^{-02}$	59.5	$0.215 \cdot 10^{-03}$
6.5	$0.268 \cdot 10^{-01}$	24.5	$0.266 \cdot 10^{-01}$	42.5	$0.131 \cdot 10^{-02}$	60.5	$0.193 \cdot 10^{-03}$
7.5	$0.322 \cdot 10^{-01}$	25.5	$0.239 \cdot 10^{-01}$	43.5	$0.103 \cdot 10^{-02}$	61.5	$0.168 \cdot 10^{-03}$
8.5	$0.368 \cdot 10^{-01}$	26.5	$0.215 \cdot 10^{-01}$	44.5	$0.817 \cdot 10^{-03}$	62.5	$0.142 \cdot 10^{-03}$
9.5	$0.405 \cdot 10^{-01}$	27.5	$0.194 \cdot 10^{-01}$	45.5	$0.653 \cdot 10^{-03}$	63.5	$0.118 \cdot 10^{-03}$
10.5	$0.434 \cdot 10^{-01}$	28.5	$0.173 \cdot 10^{-01}$	46.5	$0.535 \cdot 10^{-03}$	64.5	$0.954 \cdot 10^{-04}$
11.5	$0.456 \cdot 10^{-01}$	29.5	$0.153 \cdot 10^{-01}$	47.5	$0.465 \cdot 10^{-03}$	65.5	$0.747 \cdot 10^{-04}$
12.5	$0.472 \cdot 10^{-01}$	30.5	$0.133 \cdot 10^{-01}$	48.5	$0.433 \cdot 10^{-03}$	66.5	$0.557 \cdot 10^{-04}$
13.5	$0.483 \cdot 10^{-01}$	31.5	$0.115 \cdot 10^{-01}$	49.5	$0.419 \cdot 10^{-03}$	67.5	$0.398 \cdot 10^{-04}$
14.5	$0.488 \cdot 10^{-01}$	32.5	$0.987 \cdot 10^{-02}$	50.5	$0.405 \cdot 10^{-03}$	68.5	$0.281 \cdot 10^{-04}$
15.5	$0.487 \cdot 10^{-01}$	33.5	$0.842 \cdot 10^{-02}$	51.5	$0.386 \cdot 10^{-03}$	69.5	$0.193 \cdot 10^{-04}$
16.5	$0.479 \cdot 10^{-01}$	34.5	$0.712 \cdot 10^{-02}$	52.5	$0.368 \cdot 10^{-03}$	70.5	$0.118 \cdot 10^{-04}$
17.5	$0.466 \cdot 10^{-01}$	35.5	$0.594 \cdot 10^{-02}$	53.5	$0.356 \cdot 10^{-03}$	71.5	$0.486 \cdot 10^{-05}$

Table 7 Taylor altitude dependent velocity distribution

2.2.7.4 Flux Enhancement and Altitude Dependent Velocity Distribution

In Ref /9/ it is explained how the above velocity distribution may be adjusted to reflect its altitude dependence. The velocity correction which is used to increase the flux with decreasing distance from the Earth is used to adjust the velocity distribution which is then re-binned accordingly.

In case of a single velocity value the flux increase due to Earth gravity at a given distance r of the centre of the Earth is described by the factor G which is given by

$$G = 1 + \frac{v_{esc}^2}{v_\infty^2} \quad \text{or} \quad G = \frac{v^2}{v^2 - v_{esc}^2} ,$$

with

$$v^2 = v_{esc}^2 + v_\infty^2 .$$

Using the product μ of the constant of gravitation with Earth's mass, the escape velocity at distance r can be written as

$$v_{esc} = \sqrt{\frac{2\mu}{r}} ,$$

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and v_∞ is the velocity in free space, i.e. in the absence of Earth's gravity which is tabulated in Table 7, and v is the 'enhanced' meteoroid velocity at distance r .

To obtain the correct flux enhancement in case a velocity *distribution* is given we must realise that G is a function of v_∞ . Thus the enhanced flux F_E is obtained from the Grün flux F_G by

$$F_E = \bar{G} \cdot F_G \quad \text{with} \quad \bar{G} = \int_0^\infty n(v_\infty)G(v_\infty)dv_\infty \quad .$$

This assumes that the velocity distribution $n(v_\infty)$ has been normalised:

$$\int_0^\infty n(v_\infty)dv_\infty = 1$$

The above formulas contain the necessary information to calculate the altitude dependence of the velocity distribution, since we can write

$$\bar{G} = \int_0^\infty n(v_\infty)G(v_\infty)dv_\infty \approx \sum_{k=1}^N n_k G_k = \sum_{k=1}^N n'_k \quad ,$$

with $n_k = n(v_{\infty,k})$ and $n'_k = n'(v_k)$ representing the tabulated values for the original distribution function and for the distribution function at distance r respectively.

Given the escape velocity at distance r , v_{esc} and the tabulated values of $n(v_\infty)$ in 1 km/s bins n_k , we calculate the values n'_k for the distribution $n'(v)$ at distance r by

$$n'_k = G_k n_k$$

with

$$G_k = \frac{v_k^2}{v_k^2 + v_{esc}^2}$$

and

$$v_k = \sqrt{v_{esc}^2 + v_{\infty,k}^2} \quad .$$

If we now tabulate the values of n'_k we need to change the bin limits by inserting the values of v at the places of the given values of v_∞ which is done by using again the formula

$$v = \sqrt{v_{esc}^2 + v_\infty^2} \quad .$$

As a result the bin widths will now no longer be equidistant in v , which is the independent variable of the new distribution function $n'(v)$, so re-binning will be necessary by interpolating the values of $n'(v)$. This completes the calculation procedure of the new table for the velocity distribution $n'(v)$ at the given distance r .

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2.2.8 Particle Densities and Flux-mass Functions

2.2.8.1 Particle Densities

There is little knowledge on the densities of meteorite particles, and today's estimate has not improved over the model assumed in the existing ESABASE (Ref. /1/), which is either a user defined constant with a default density of $\rho = 0.5 \text{ g/cm}^3$, or it is calculated by the following decreasing and discontinuous function of the meteoroid mass m :

$$\rho (m) = 2.0 \text{ g/cm}^3 \quad \text{for} \quad m < 10^{-6} \text{ g}$$

$$\rho (m) = 1.0 \text{ g/cm}^3 \quad \text{for} \quad 10^{-6} \text{ g} \leq m \leq 10^{-2} \text{ g}$$

$$\rho (m) = 0.5 \text{ g/cm}^3 \quad \text{for} \quad m > 10^{-2} \text{ g}$$

2.2.8.2 Flux-size to Flux-mass Function

In some cases it will be necessary to convert a flux which is given by its $F(d)$ function to the $F(m)$ function or vice versa. With the meteoroid mass density ρ and its particle diameter d and an assumed spherical shape the relation

$$m = 1/6 \rho \pi d^3$$

is used.

2.2.9 Earth Shielding and Gravitational Effects

A description of the consideration of gravitational focussing and Earth shielding effects of the omni-directional meteoroid models is given in the following subsections.

Within the Divine-Staubach model these effects are treated as given in /20/ or /22/. The corresponding equations are given in section 2.2.3.1.

2.2.9.1 Gravitational Focusing

With the exception of the streams contributions of Jenniskens, which are derived from measurements at the meteoroids entry into the Earth's atmosphere, the fluxes are given in the models at a distance of 1 AU from the Sun at the Earth's position but in absence of the Earth. Thus the change of the particle trajectories due to Earth attraction needs to be taken into account. This will change the flux by a flux increase factor which is denoted by $G_e(h)$ and is a function of the target altitude h above the Earth surface.

When the Grün sporadic flux model is used, the flux is corrected by the factor

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$$G_e(h) = 1 + \frac{R_e + 100}{R_e + h} ,$$

which is given in Ref. /3/. $R_e + 100 = 6478$ km, which is the Earth radius augmented by 100 km atmosphere height.

If the altitude dependent velocity distribution of section 2.2.7.3 is used, the gravitational flux increase must be calculated as described in 2.2.7.4 .

2.2.9.2 Earth Shielding

Earth shielding is accounted for by excluding all arrival directions which are within an angle θ subtended with the direction of the Earth's centre, the 'Earth-shielding cone'. By subtracting the corresponding solid angle element from the unit sphere the shielding factor η

$$\eta = \frac{1 + \cos\theta}{2}$$

is obtained for the case of a *randomly tumbling* plate. The angle θ is then geometrically given by

$$\sin\theta = \frac{R_e + 100}{R_e + h}$$

where R_e is the mean Earth radius in km and h is the target orbiter altitude in km. The atmosphere height is accounted for by the constant 100 km.

Taking into account that all the meteoroid orbits which have velocities higher than the escape velocity at the Earth's atmosphere will be 'seen' at the top of the atmosphere, the determination of the angle θ needs to be made in a more precise way, by using the formula

$$v = \sqrt{v_{esc}^2 + v_{\infty}^2}$$

which was mentioned in section 2.2.7.4. This yields a modified expression for the angle θ

$$\sin\theta = \frac{(R_e + 100)\sqrt{v_{\infty}^2 + v_{esc}^2}(R_e + 100)}{(R_e + h)\sqrt{v_{\infty}^2 + v_{esc}^2}(R_e + h)}$$

which gives 5% to 10% better results. If a velocity distribution is used, the mean value

$$\bar{\theta} = \int_0^{\infty} n(v_{\infty})\theta(v_{\infty})dv_{\infty}$$

needs to be calculated by integrating over the normalised velocity distribution $n(v_{\infty})$.

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For an oriented plate the above relation between η and θ is no longer correct. The collision probability is then strongly dependent on the angle between the normal to the plate and the Earth direction. This problem can be geometrically formulated, but unfortunately the solution becomes unwieldy in the general case and contains an elliptical integral at one place. For the strict application of the ray tracing technique this is however not a problem, because only the angle θ (and in some cases possibly $\bar{\theta}$) will be used to determine if a ray must be fired or not in a certain direction.

2.2.10 Ray Tracing and k-Factor

As it has been discussed in the previous sections, most of the particle fluxes which are calculated with the presented models are quantified with respect to a "randomly tumbling plate" or even a "virtual randomly tumbling plate which is stationary with respect to the Earth's surface".

The analysis of the debris / micrometeoroid environment hazard acting upon a spacecraft requires the computation of particle fluxes on oriented surfaces. The method chosen for this uses a ray tracing technique, which enables to account for the mutual shading between different surfaces of the orbiting structure.

The relative relation between the flux on a virtual randomly tumbling plate which is stationary with respect to the Earth surface above Earth, the flux on a randomly tumbling plate in orbit and the flux on an oriented plate in orbit has been discussed in several papers in the past.

In the review phase of the study, the topic was thoroughly investigated, also with respect to the implementation of the ray tracing technique for the analysis in the enhanced ESABASE/ Debris tool. The analytical assessment was backed up by numerical analysis using ray tracing. This work is documented in Annex C of (Ref. /15/).

The main finding in Ref. /15/, Annex C is that a properly implemented ray tracing fully renders the true situation, without the necessity to include an additional k factor. This result also encourages the use of ray tracing to account for the Earth shielding, as it has been lined out in paragraph 2.2.7.2.

An abstract of the investigation documented in (Ref. /15/) is included in Annex A.

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3 The Damage Equations

This chapter describes the equations used for the modelling of the interaction between micro-particles and satellite structures. The satellite structure can be either a single wall (e.g. aluminium) or a multiple wall, typically in the case when a specific micro-particle shielding or thermal protection (MLI) is applied to the basic structure.

Due to the increasing concern of the risk posed by the micro-particle environment of long term missions (above all the international space station), special micro-particle shields have been developed.

The particle / wall interaction models - in this document referred to as damage equations - describe the phenomena of high and hyper velocity impacts on structures. The typical impact velocity for space debris is 8 to 10 km/s, for meteoroids about 20 km/s. The equations are largely derived from experiments.

The damage equations are treated in two separate groups:

- The *ballistic limit equations*, which yield the critical impacting particle size above which the structure fails. Different equations are used for single and multiple wall structures.
- The *damage size equations*, which yield the crater size of semi-infinite targets and the hole diameter of punctured targets (generally thin walls).

In this chapter, the 5 classes of damage equations which are implemented in the ESABASE2/ DEBRIS analysis tool are described:

- *The single wall ballistic limit equation*
- *The multiple wall ballistic limit equation*
- *The crater size equation*
- *The generic clear hole equation*
- *The advanced hole equation*

ESABASE2/Debris further offers the option to integrate a user defined subroutine for the damage equation. This requires a FORTRAN 77 compiler and linker and is only advised for the advanced user and hyper velocity impact expert. For details, refer to (Ref. /14/).

3.1 The Parametric Formulation of the Damage Equations

To provide the necessary flexibility in the usage of currently available and possible future damage equation formulations, the 5 classes of damage equations have been formulated in a parametric form, allowing the user to adapt the equation to his needs.

It must be clearly stated however, that parameter variations of the damage equations should only be performed by the experienced user and hypervelocity impact specialist. For the beginner, the standard equations, activated by key words in the software, are recommended, see (Ref. /14/).

The parameters of the standard equations of each class are tabled in the respective sub chapters.

In the software, the ballistic limit equations are used to compute the critical particle diameter, and the damage size equations to compute the crater or hole diameter. The damage equations are presented in these formulations in the following sub-sections.

In the equations, the following general symbols are used:

Symbol	Unit	Description
t_t, t_B, t_s	[cm]	Thickness of Target, Back-up wall, Shield
K	[-]	Characteristic Factor
d_p	[cm]	Particle (impactor) Diameter
$\rho_t, \rho_p, \rho_s, \rho_B$	[g/cm ³]	Density of Target, Particle, Shield, Back-up wall
v	[km/s]	Impact velocity
α	[--]	Impact angle
S	[cm]	Spacing between shielding and back-up wall
D	[cm]	Crater or Hole diameter
F_{mx}	[cm]	Ballistic Limit

3.2 The Single Wall Ballistic Limit Equation

The parametric formulation of the equation is

$$d_{p,lim} = \left[\frac{t_t}{K_f \cdot K_1 \cdot \rho_p^\beta \cdot v^\gamma \cdot (\cos \alpha)^\xi \cdot \rho_t^\kappa} \right]^{\frac{1}{\lambda}}$$

The K_f factor allows specifying what type of damage is considered a failure for the ESABASE2 thick plate equation and the glass target equations. In the other equations it is not used. The K_1 factor includes other parameters particular to each of equation (e.g. target yield strength σ_t).

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It is also often found in the following form:

$$F_{mx} = K_o \cdot K_{mat} \cdot d_p^\lambda \cdot \rho_p^\beta \cdot v^\gamma \cdot (\cos \alpha)^\xi \cdot \rho_t^\kappa$$

The K_{mat} factor is material dependent. Compared to the first formulation, $K_o \cdot K_{mat} = K_1$. In the second formulation, the failure specification does not appear explicitly.

The values of the parameters for standard equations are given below. For this group of equations, the yield strength σ_t of the target material is to be input in ksi.

Equation	K_f ¹⁾	K_1 ²⁾	λ	β	γ	ξ	κ
ESABASE Thick Plate	1.8 ÷ 3	0.2 ÷ 0.33	1.056	0.519	2/3	2/3	0
ESABASE Thin Plate	1.0	0.26 ÷ 0.64	1.056	0.519	0.875	0.875	0
MLI ³⁾	1.0	0.37	1.056	0.519	0.875	0.875	0
Pailer-Grün	1.0	0.77	1.212	0.737	0.875	0.875	-0.5
McDonnell & Sullivan	1.0	$0.756 \left[\frac{\sigma_{Al}}{\sigma_t} \right]^{0.134}$	1.056	0.476	0.806	0.806	-0.476
Gardner	1.0	$0.608 \sigma_t^{-0.093}$	1.059	0.686	0.976	0.976	-0.343
Gardner, McDonnell, Collier	1.0	$0.85 \sigma_t^{-0.153}$	1.056	0.763	0.763	0.763	-0.382
Frost	1.0	0.43	1.056	0.519	0.875	0.875	0
Naumann, Jex, Johnson	1.0	0.65	1.056	0.5	0.875	0.875	-0.5
Naumann	1.0	0.326	1.056	0.499	2/3	2/3	0
McHugh & Richardson Thick glass target	1.85 ÷ 7	0.64	1.2	0	2/3	2/3	0.5
Cour-Palais Thick glass target	1.85 ÷ 7	0.53	1.06	0.5	2/3	2/3	0

Table 8 Single wall ballistic limit equation typical parameter values. In this table, all yield strengths are assumed to be given in ksi.

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Notes

1) Failure factors K_f :

- ESABASE Thick Plate:

$K_f \geq 3$	Crater generation without spall
$2.2 \leq K_f < 3$	Spallation of the plate
$1.8 \leq K_f < 2.2$	Spall breaks away
$K_f < 1.8$	Perforation of the plate
- Thick Glass Targets:

$K_f \geq 7$	Crater generation without spall
$1.85 \leq K_f < 7$	Spallation of the plate
$K_f < 1.85$	Perforation of the plate

2) K_1 factors:

- ESABASE Thick Plate:

Aluminium alloys:	$K_1 = 0.33$
Stainless steel:	$K_1 = 0.2$
- ESABASE Thin Plate:

Aluminium alloys:	$K_1 = 0.43 - 0.454$
Stainless steel:	$K_1 = 0.255$ AISI 304, AISI 306
	$K_1 = 0.302$ 17-4 PH annealed
Magnesium lithium:	$K_1 = 0.637$
Columbium alloys:	$K_1 = 0.271$
- McDonnell & Sullivan: Reference σ_t -values are given in Table 9
- Gardner: σ_t shall be input in Pa for this equation

3) The single wall ballistic limit equation for MLI assesses the failure of the thermal blanket, and was derived by tests and hydro-code simulations using the ESABASE thin plate equation as starting point, see ref. /2/. The equation is expressed as

$$F_{mx} = 0.37 \cdot d_p^{1.056} \cdot \rho_p^{0.519} \cdot (v \cdot \cos \alpha)^{0.875}$$

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Some reference values for the 0.2 yield strength σ is given below (used in the McDonnell & Sullivan and Gardner equations):

Material	[ksi] ¹⁾	[MPa]
Aluminium pure	10	70
Aluminium alloys (superior)	30 - 65	200 - 450
Silver	22	150
Gold	17.5	120
Beryllium copper	120	830
Copper	32	220
Stainless steel	110	760
Titanium	140	980

Table 9 Some values of yield strength

Note: ¹⁾ ksi = kilo.lb/sq.-inch = 1000 lb / inch² = 6.895 MPa

3.3 The Multiple Wall Ballistic Limit Equation

The parametric form of this equation is

$$d_{p,\text{lim}} = \left[\frac{t_B + K_2 \cdot t_s^\mu \cdot \rho_s^{v_2}}{K_1 \cdot \rho_p^\beta \cdot v^\gamma \cdot (\cos \alpha)^\xi \cdot \rho_B^\kappa \cdot S^\delta \cdot \rho_s^{v_1}} \right]^{\frac{1}{\lambda}}$$

It is also often formulated as:

$$F_{mx} = K_1 \cdot d_p^\lambda \cdot \rho_p^\beta \cdot v^\gamma \cdot [\cos \alpha]^\xi \cdot \rho_B^\kappa \cdot S^\delta \cdot \rho_s^{v_1} - K_2 \cdot t_s^\mu \cdot \rho_s^{v_2}$$

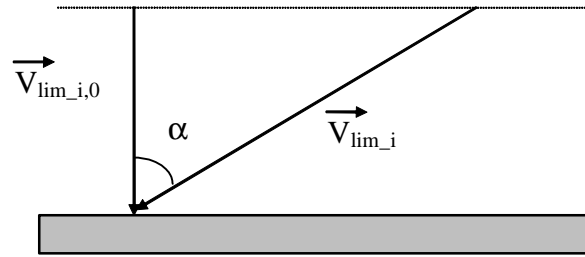
Three velocity regions are defined, delimited by the two limit velocities $v_{\text{lim}1}$ and $v_{\text{lim}2}$. The governing parameters mostly have different values for velocities below $v_{\text{lim}1}$ and above $v_{\text{lim}2}$. For velocities between $v_{\text{lim}1}$ and $v_{\text{lim}2}$, a linear interpolation is performed.

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The limit velocities may vary with the impact angle:

$$v_{\text{lim}1} = v_{\text{lim}1,0} \cdot (\cos \alpha)^{\varphi_1}$$

$$v_{\text{lim}2} = v_{\text{lim}2,0} \cdot (\cos \alpha)^{\varphi_2}$$



Where the normal velocity component is used (which is generally the case), i.e. $\gamma = \xi$, the cosine exponent in the equations above is $\varphi_1 = \varphi_2 = -1$.

The values of the parameters for typical multiple wall equations are given in Table 8 and Table 11 below (here the limit velocities are defined as function of the impact angle).

For the Cour-Palais, MLI and Maiden-McMillan equations, only one velocity domain is used.

Equation	K_1	K_2	λ	β	γ	κ	δ	ξ	v_1/v_2	μ	
Cour-Palais	$0.044 \left[\frac{\sigma_{y,ref}}{\sigma_{y,t}} \right]^{0.5}$	0	1	0.5	1	0.167	-0.5	1	0 / 0	0	
MLI ³⁾	$0.034 \left[\frac{\sigma_{y,ref}}{\sigma_{y,t}} \right]^{0.5}$	0	1	0.5	1	0.167	-0.5	1	0 / 0	0	
Maiden-McMillan ¹⁾	$K_f \cdot \frac{\pi}{6} \left[\frac{\sigma_{y,ref}}{\sigma_{y,t}} \right]^{0.5}$	0	3	1	1	0	-2	1	0 / 0	0	
ESA ²⁾	$v < 3$	0.255 ÷ 0.637	1	1.056	0.519	0.875	0	0	0.875	0 / 0	1
	$v > 9.5$	$\frac{\pi}{6} \left[\frac{\sigma_{y,ref}}{\sigma_{y,t}} \right]^{0.5}$	0	3	1	1	0	0	1	0 / 0	0

Table 10 Standard Double wall ballistic limit equation parameter values

Notes

¹⁾ Failure factors K_f for the Maiden-McMillan equation:

$K_f \geq 41.5$	No damage
$8.2 \leq K_f < 41.5$	Incipient yield zone
$7.1 \leq K_f < 8.2$	Fracture zone

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$K_f < 7.1$

Penetration zone

The K_f factor flows into the K_1 factor, see Table 10.

2) *ESA Equation:*

- The Boeing-ESA equation described in chapters 1 and 2 has the same form as the ESA equation, but with $v_{lim1} = 1.4$ km/s and $v_{lim2} = 7.83$ km/s.
- The reference yield strength $\sigma_{y,ref} = 70'000$ lb/in² = 482.8 MPa.

3) *MLI Equation*

The multiple wall ballistic limit equation for MLI assesses the debris / meteoroid protection of the thermal blanket, and was derived by tests and hydro-code simulations using the Cour-Palais equation as starting point, see ref. /2/.

The equation is expressed as $F_{mx} = 0.034 \cdot \left(\frac{70000}{\sigma_w}\right)^{0.5} \cdot \rho_p^{0.5} \cdot \rho_w^{0.167} \cdot d_p \cdot (v \cdot \cos \alpha) \cdot S^{-0.5}$ with σ_w in lb/in².

Equation		K_1	K_2	λ	β	γ	κ	δ	ξ	v_1/v_2	μ
ESA	$v < 3$ km/s	$0.312(\tau_1^*/\tau)^{0.5}$	$1.667 \cdot K_1$	1.056	0.5	2/3	0	0	5/3	0/0	1
Triple	$v > 7$ km/s	$0.107(\tau_1^*/\tau)^{0.5}$	0	1.5	0.5	1	0	-0.5	1	0.167/0	0
NASA	$v < 3$ km/s	$0.6 (\sigma_w/40)^{-0.5}$	$(\sigma_w^*/40)^{-0.5}$	1.056	0.5	2/3	0	0	5/3	0/0	1
ISS	$v > 7$ km/s	$[3.918(\sigma_w/70)^{1/3}]^{-1.5}$	0	1.5	0.5	1	0	-0.5	1	0.167/0	0
NASA	$v < 3$ km/s	$0.3 (\tau_1^*/\tau)^{0.5}$	$1.233 \cdot K_1$	1.056	0.5	2/3	0	0	5/3	0/1	1
Shock	$v > 6$ km/s	$22.545(\tau_1^*/\tau)^{0.5}$	0	3	1	1	-1	-2	1	0/0	0
NASA	$v < 3$ km/s	$0.4(\tau_1^*/\tau)^{0.5}$	$0.925 \cdot K_1$	1.056	0.5	2/3	0	0	5/3	0/1	1
Bumper	$v > 6$ km/s	$18.224(\tau_1^*/\tau)^{0.5}$	0	3	1	1	-1	-2	1	0/0	0

Table 11 Standard Multiple wall ballistic limit equation parameter values

Notes: τ_1^* is the yield stress of a reference material (higher quality aluminium)

$\tau_1^* = 40'000$ lb/in² = 276E6 Pa.

$\sigma_w = 47$ ksi for the reference equation used for system tests. σ_w to be input in ksi

3.4 The Crater Size Equation

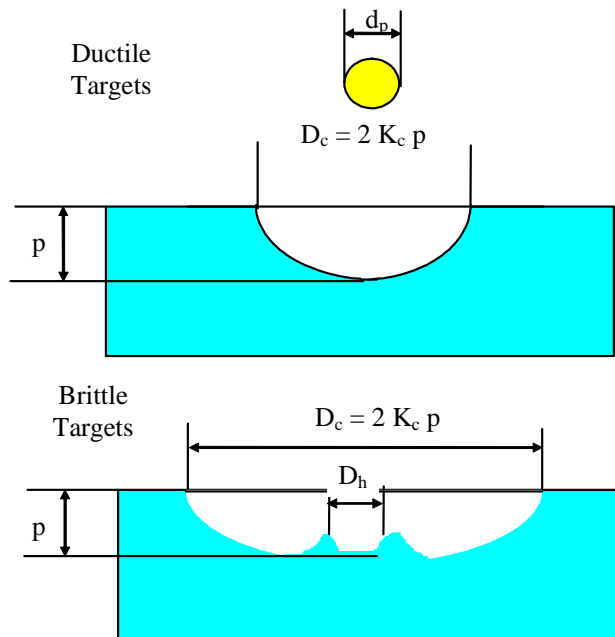
The parametric form of the equation is:

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$$D = K_1 \cdot K_c \cdot d_p^\lambda \cdot \rho_p^\beta \cdot v^\gamma \cdot (\cos \alpha)^\xi \cdot \rho_t^\kappa$$

It is basically the same as that of the single wall ballistic limit equation, see section 3.2.

The crater factor K_c is the ratio of the crater radius $D/2$ to the crater depth p , see sketch.



Strictly speaking, the crater size equation is only valid when no failure occurs.

For ductile targets, the crater is more or less spherical, and $K_c \approx 1$.

For brittle targets, an interior crater with diameter D_h may form, the outer crater (with diameter D_c) being much larger. For brittle targets, K_c may be as high as 10.

The crater size equation assumes a semi-infinite target and should only be used for cases where the wall thickness is much larger than the particle diameter.

The values of the parameters for typical equations are given in Table 12.

Equation	K_c	K_1	λ	β	γ	ξ	κ
Ductile targets							
ESABASE Thick Plate ¹⁾	2	0.4 ÷ 0.66	1.056	0.519	2/3	2/3	0
Shanbing et al	$n/d^{2)}$	$0.54 (\sigma_{y,t})^{-1/3}$	1	2/3	2/3	2/3	-1/3
Sorensen	$n/d^{2)}$	$0.622 (\tau_t)^{-0.282}$	1	0.167	0.564	0.564	0.115
Christiansen for $\frac{\rho_p}{\rho_t} < 1.5$	$n/d^{2)}$	$10.5 H_t^{1/4} \cdot c_s^{2/3}$	1.056	0.5	2/3	2/3	-0.5

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Equation	K_c	K_1	λ	β	γ	ξ	κ
Christiansen for $\frac{\rho_p}{\rho_t} > 1.5$	n/d ²⁾	$10.5 H_t^{-\frac{1}{4}} \cdot c_s^{\frac{2}{3}}$	1.056	2/3	2/3	2/3	-2/3
Brittle targets							
Gault	n/d ²⁾	1.08	1.071	0.524	0.714	0.714	-0.5
Fechtig	n/d ²⁾	6.0	1.13	0.71	0.755	0.755	-0.5
McHugh & Richardson	n/d ²⁾	1.28	1.2	0	2/3	2/3	0.5
Cour-Palais	n/d ²⁾	1.06	1.06	0.5	2/3	2/3	0

Table 12 Standard Crater size equation parameter values

Notes

1) K_1 factors:

- ESABASE Thick Plate: Aluminium alloys: $K_1 = 0.66$
Stainless steel: $K_1 = 0.4$

2) "n/d" means not defined in the equation reference.

The software uses a default value of 1 for ductile targets and 10 for brittle targets.

3) Christiansen equations

- H_t is the target Brinell hardness. A typical value is 90.
- c_s is the velocity of sound in the target material. For steel, $c_s = 5.85$ km/s.

3.5 The Generic Clear Hole Equation

The parametric form of the equation is:

$$D = \left\{ K_0 \cdot \left(\frac{t_s}{d_p} \right)^\lambda \cdot \rho_p^\beta \cdot v^\gamma \cdot (\cos \alpha)^\xi \cdot \rho_s^\nu + A \right\} \cdot d_p$$

The clear hole equation is only valid for a full perforation, i.e. mainly for thin foils (typically bumper shields or similar).

The limit of validity is given by the relation $\frac{t_s}{d_p} < 10$

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The values of the parameters for the common standard equations are given in Table 13 below:

Equation	K_0	λ	β	γ	ξ	ν	A
Maiden	0.88	2/3	0	1	1	0	+0.9
Nysmith-Denardo	0.88	0.45	0.5	0.5	0.5	0	0
Sawle	0.209	2/3	0.2	0.2	0.2	-0.2	+1
Fechtig	5.24E-5	0	1/3	2/3	2/3	0	0

Table 13 Standard Classical hole size equation parameter values

3.6 The Advanced Hole Equation

The advanced form of the hole size equation is based on the equations derived at UniSpace Kent by Dr. Gardner, Prof. McDonnell and Dr. Collier.

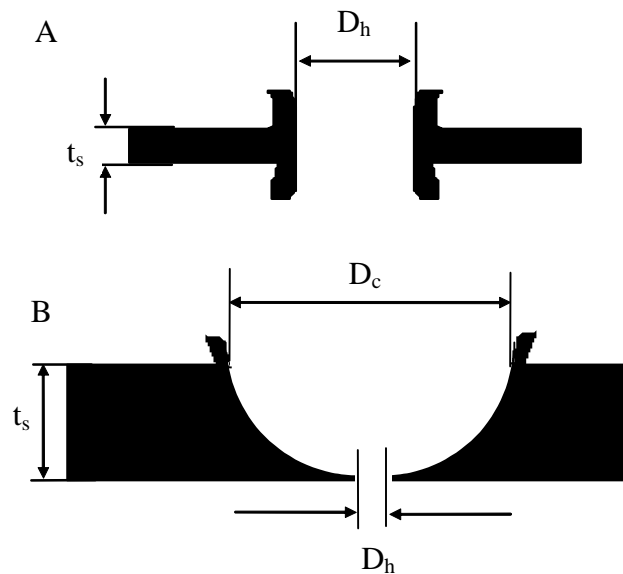
The equation is derived for the computation of the particle size from the impact velocity and the hole size on the back side of the target shield.

The equation is only valid for ductile targets. Dimensionless hole and particle diameters are used:

$$d'_p = \frac{d_p}{t_s}$$

$$D'_h = \frac{D_h}{t_s}$$

d_p is the particle diameter, D_h is the perforated hole diameter, t_s is the target thickness.



The general form of the equation is

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$$d'_p = A \cdot \left(\frac{10}{9 + \exp\left[\frac{D'_h}{B}\right]} \right) + D'_h \left(1 - \exp\left[-\frac{D'_h}{B}\right] \right)$$

$$A = \frac{d_p}{F_{mx}} = 6.97 \cdot \left(\frac{V_n \cdot \rho_p}{\sqrt{\sigma_s \cdot \rho_s}} \right)^{-0.723} \cdot \left(\frac{\sigma_s}{\sigma_{Al}} \right)^{-0.217} \cdot t_s^{-0.053}$$

F_{mx} is the ballistic limit, $V_n = V \cdot \cos\alpha$ the normal impact velocity, ρ_p the particle density, t_s , ρ_s and σ_s the target thickness, density and yield stress. σ_{Al} is the yield stress of Aluminium.

In the above equation, all units are uniform (e.g. SI), except for t_s , which is in μm .

An alternative form of the equation of A is:

$$A = 2.35 \cdot \left(\frac{V_n \cdot \rho_p}{\sqrt{\rho_s}} \right)^{-0.723} \cdot \sigma_s^{0.145} \cdot t_s^{-0.053}$$

with V_n in km/s, ρ in kg/m^3 , σ in Pa and t_s in μm

Using the standard units of ESABASE/Debris, i.e. km/s for velocity, cm for thickness, g/cm^3 for densities and MPa for stresses, the constant of the second equation for A above becomes 0.88 (in the place of 2.35).

$$B = B_1 + B_2 \cdot V_n = B_1 + B_2 \cdot V \cdot \cos \alpha$$

B_1 and B_2 are taken from Table 14.

The equation for A can be used as a ballistic limit equation.

For the computation of the hole size in the target or shield, the basic equation cannot be used directly, since it is a function of $d_p(D_h)$. The form of this equation does not allow an analytical inversion. Thus a numerical scheme must be used (e.g. Newton method).

For the starting value of D_h , the Carey, McDonnell, Dixon equation is used:

$$\frac{D_h}{d_p} = 1 + 2.9 \left(\frac{\rho_s}{\rho_p} \right)^{0.6} \cdot \left(\frac{t_s}{d_p} \right) \cdot V_n \left[\frac{1}{1 + 2.9 \left(\frac{\rho_s}{\rho_p} \right)^{0.6} \cdot \left(\frac{t_s}{d_p} \right)^2 \cdot V_n^{-m}} \right]$$

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$$V_n = V \cdot \cos \alpha$$

$$\text{For } 2 < V_n < 20 \text{ km/s: } 1.02 - 4 \cdot \exp(-0.9V_n^{0.9}) - 0.003(20 - V_n)$$

$$\text{For } V_n \geq 20 \text{ km/s: } m = 1.02$$

The equations for the parameters A and B can be parametrised as follows:

$$A = K_1 \cdot V^\gamma \cdot (\cos \alpha)^\xi \cdot \rho_p^\beta \cdot \rho_s^\nu \cdot t_s^\lambda$$

$$B = B_1 + B_2 \cdot V \cdot (\cos \alpha)^u$$

$$K_1 = 0.88 \cdot \sigma_s^{0.145}, \gamma = -0.723, \xi = -0.723, \beta = -0.723, \nu = 0.362, \lambda = -0.053$$

B_1 and B_2 are to be taken from Table 14, $u = 1 + (\xi - \gamma)$.

For more flexibility, one could also use the formulation $V_n = V \cdot (\cos \alpha)^u$ in the Carey, McDonnell, Dixon equation.

Material		Velocity V_n (km/s)	Density ρ_s (kg/m ³)	Yield stress σ_s (MPa)	B_1 (-)	B_2 (s/km)
Al	Aluminium	1.2 - 6.0	2780	69	-0.004	1.85
Al	Aluminium	6.0 - 10.7	2780	69	6.66	0.74
Ag	Silver	2.9 - 5.6	10500	150	7.92	3.14
Au	Gold	2.1 - 7.5	19300	120	6.65	2.77
BeCu	Beryllium Copper	3.7 - 6.4	8240	828	-26.3	10.3
Cu	Copper	2.0 - 6.9	8950	220	3.2	2.62
SS	Stainless Steel	2.2 - 3.7	7840	759	0.11	2.34
Ti	Titanium	2.3 - 6.6	4720	986	0.618	2.26

Table 14 Typical values of B_1 and B_2

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Note: The experimental data has shown a rather important scatter for the samples of material other than aluminium. For velocity regions outside the given ranges, the 0.74 figure for aluminium may also be used. Of the AI parameter sets, only the one in the lower velocity regime is implemented in the software. For the higher velocity regime, the user specified input option has to be used.

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4 The Secondary Ejecta Model

This chapter describes the secondary ejecta module which has been implemented in the ESABASE2/ DEBRIS analysis tool. This model permits to evaluate the effect of ejecta produced by primary impacts (from space debris or meteoroids) on surrounding faces of the analysed structure.

4.1 Ejecta Phenomenon

4.1.1 Normal Impacts

Material ejection under hypervelocity impact is divided in three processes corresponding to different physical and mechanical phenomena: jetting phase, debris cone formation and spallation.

In general, no spalls are observed on ductile targets.

The jetting phase:

During the first times of projectile penetrating into the target, both target and projectile undergo partial or complete melting and vaporisation. A certain amount of material is ejected from the impact interface. The physical state of the ejected material is mainly liquid and the ejection angle is approximately 20° measured from the target surface. The ratio of jetted mass to total ejected mass is very small, less than 1%.

The debris cone:

Later in the crater formation, the target material is finely commuted in fine solid fragments by compression or tensile failure, these fragments are ejected in a thin debris cone. The physical state of the ejected material is mainly solid. The ejection angle is between 60° and 80° measured from the target surface and depends on target characteristics. The ratio of ejected mass to total ejected mass is estimated between 50% and 70%. The ejection velocity from a few m/s to a few km/s is inversely proportional to fragment size. The minimum size depends mainly on target characteristics and should be sub-micron sized. The maximum size can be evaluated by empirical relations. The size distribution is inversely proportional to the square of the fragment size.

The spallation phase:

In general, no spallation is observed on ductile targets.

Near the free surface, rarefaction waves produce tensile stress. In brittle material, tensile failure leads to the formation of spall fragments that are ejected. The physical state of the ejected material is mainly solid. The ejection direction is normal to the surface.

The ejection velocity is less than 1 km/s and is 10 to 100 times less than impact velocity. The fragment size is large, about 10 times the size of debris cone fragments. These fragments have plate shape whose dimensions are difficult to evaluate because large plate fragments are likely to fragment themselves into smaller particles. The ratio of spalled mass total mass is estimated between 30% and 50%.

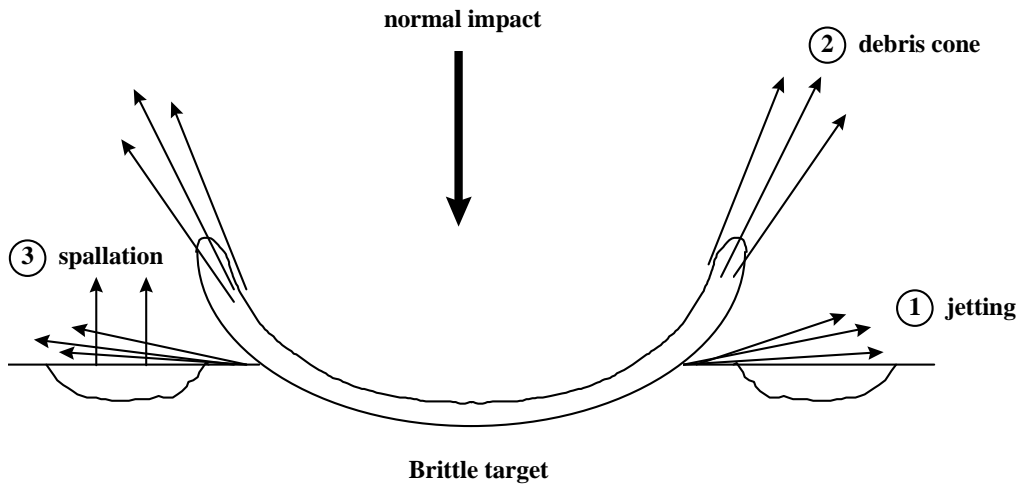


Figure 4-1 Schematic summary of ejection processes under normal impact.

4.1.2 Oblique Impacts

Oblique impacts with impact angles $> 60^\circ$ (measured from the target surface normal) are considered separately.

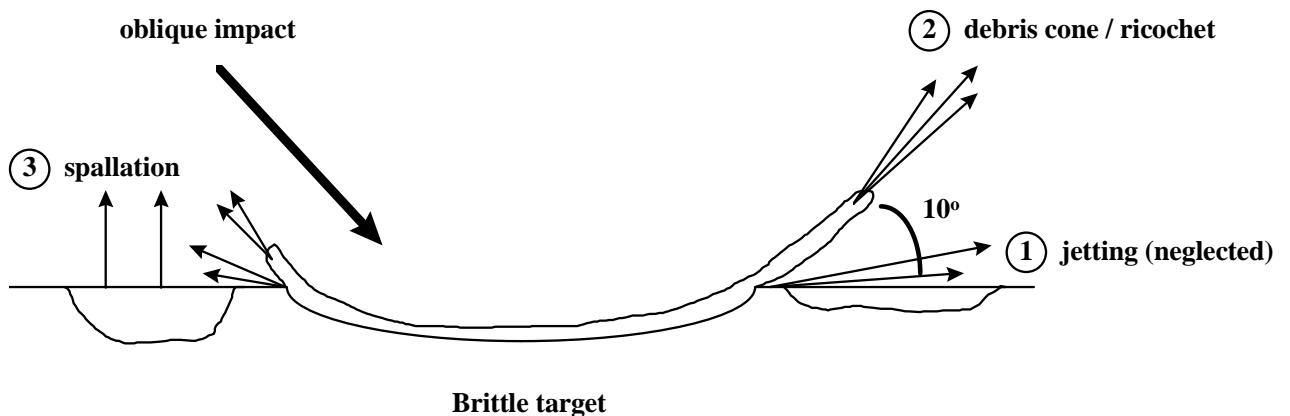


Figure 4-2 Schematic summary of ejection processes under oblique impact.

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The phenomena involved in ejecta formation are similar to those observed for normal impacts: jetting phase, debris cone formation and spallation phase.

The main differences are:

- the total ejected mass decreases with decreasing impact angle,
- the debris cone central axis remains vertical but the cone is flattened. The debris cone ejection angle decreases with decreasing incidence angle,
- the problem is not any more axisymmetric as the obliquity of impact favours the azimuth direction downstream the incidence angle.

4.2 Enhanced Ejecta Model

4.2.1 General Description

The ejecta model distinguishes between ductile and brittle targets. Calculations are made in the satellite frame. Therefore, the target is considered as immobile and the impact velocity is the relative velocity between the projectile and the target surface.

The *model inputs* are then:

- target characteristics: ductile or brittle, density (ρ_t) ;
- projectile characteristics: mass (m_i), density (ρ_i) ;
- impact characteristics: relative impact velocity vector determined by the scalar velocity (v_i) and the impact direction (φ_i, θ_i).

A description of the model input is given in section 4.2.2.2.

The *model outputs* consist of an analytical function providing the number $n(\varphi, \theta, \delta, v)$ of fragments of size (δ) ejected at velocity (v) in the spatial direction (φ, θ), by solid angle unity. It is basically assumed that $n(\varphi, \theta, \delta, v)$ is the sum of independent terms corresponding to the different ejection processes:

- $n_{\text{cone}}(\varphi, \theta, \delta, v) + n_{\text{spalls}}(\varphi, \theta, \delta, v)$ for brittle targets ;
- $n_{\text{cone}}(\varphi, \theta, \delta, v)$ only for ductile targets.

The *jetting phenomenon is neglected* since the proportion of material involved is less than 1% of the total ejected mass.

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The model (Ref. /16/) provides a continuous ejecta distribution in the geometrical space around the impact and has thus to be adapted for an implementation into ESABASE/Debris. The recent upgrade of the ejecta model as described in the following sections considers the conservation of the momentum and the energy of the impacting particle and the ejected particles, as well as the latest upgrade of the mathematical theory (Ref. /24/).

4.2.2 Software Model

The introduction of an ejecta model into ESABASE2/Debris allows assessing the influence of secondary impacts in terms of flux and penetration. The ejected fragments of material resulting from the particle primary impact shall thus be treated in the same way as the primary particle impacting the surface and thus shall be characterised by the same parameters:

- particle size (or mass),
- number of particles,
- particle density,
- particle velocity.

4.2.2.1 Description of the Ejecta model

4.2.2.1.1 Total ejected mass

The basic equation is taken from Gault (1973), valid for brittle target and for an incident particle with a diameter larger than 10 μm :

$$\theta_i \leq 60^\circ: \quad M_{Gault} = 7.41 \cdot 10^{-6} \sqrt{\frac{\rho_p}{\rho_t}} E_i^{1.133} (\cos \theta_i)^2 \quad (\text{SI units})$$

$$\theta_i > 60^\circ: \quad M_{Gault} = 7.41 \cdot 10^{-6} \sqrt{\frac{\rho_p}{\rho_t}} E_i^{1.133} (\cos 60^\circ)^2 \quad (\text{SI units})$$

with: E_i impact kinetic energy
 ρ_p projectile density
 ρ_t target density
 θ_i impact incidence angle (from normal direction)

For other cases, we introduce a corrective coefficient K :

$$M_e = K M_{Gault}$$

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	$d_p < 10 \mu\text{m}$	$d_p \geq 10 \mu\text{m}$
ductile target	$K = 10^{-2} - 10^{-3}$	
brittle target	$K = d_p/10^5$	$K = 1$

with: d_p projectile diameter

4.2.2.1.2 Mass partitioning

The total ejected mass is partitioned in three components:

$$M_e = M_{jet} + M_{cone} + M_{spalls}$$

$$M_{jet} \approx 0$$

$$M_{cone} = \beta M_e$$

$$M_{spalls} = (1 - \beta) M_e$$

	$d_p \leq 1 \mu\text{m}$	$1 \mu\text{m} < d_p \leq 10 \mu\text{m}$	$10 \mu\text{m} < d_p \leq 100 \mu\text{m}$	$d_p > 100 \mu\text{m}$
ductile target	$\beta = 1$			
brittle target	$\beta = 1$	$\beta = -0.3 \log d_p - 0.8$		$\beta = 0.4$
solar cell	$\beta = 1$	$\beta = -0.3 \log d_p - 0.8$	$\beta = -0.6 \log d_p - 2.3$	$\beta = 0.1$

4.2.2.1.3 Cone Fragments Modelling

For each primary particle with a velocity v_i , an impact direction (θ_i, φ_i) and a size (δ_i) , the debris cone and secondary particle impacts are modelled using analytical formulation and ray-tracing technique. The software model calculates the number of secondary particle $(n(\varphi, \theta, \delta, v))$ in function of size $(\delta, \text{in } N_\delta \text{ intervals})$ and the velocity (v) in randomly distributed directions. Each direction, characterising a ray, is determined by its elevation (θ) and its azimuth (φ) . The number of rays in $(\theta \in [0, \pi/2], \varphi \in [0, 2\pi])$ is fixed by the user.

We suppose that δ (ejecta diameter), θ (ejecta zenith) and φ (ejecta azimuth) are independent variables and that V (ejecta velocity) is a function of these 3 variables:

$$n_{cone}(\delta, \theta, \varphi, V) = K_{cone} f_{cone}(\delta) g_{cone}(\theta) h_{cone}(\varphi) \Delta(V - V_{cone}(\delta, \theta, \varphi))$$

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Size distribution

$$f_{cone}(\delta) = \frac{1-\alpha_1}{\delta_{max}^{1-\alpha_2} \delta_1^{\alpha_2-\alpha_1} - \delta_{min}^{1-\alpha_1}} \delta^{-\alpha_1} \mathbf{1}[\delta_{min}, \delta_1] + \frac{1-\alpha_2}{\delta_{max}^{1-\alpha_2} - \delta_{min}^{1-\alpha_1} \delta_1^{\alpha_1-\alpha_2}} \delta^{-\alpha_2} \mathbf{1}[\delta_1, \delta_{max}]$$

$$\delta_{min} = 0.1 \mu m$$

$$\delta_{max} = \sqrt[3]{\frac{6 m_b}{\pi \rho_t}}$$

$$m_b = \lambda M_e$$

if $\delta_{max} > 10 \mu m$ $\delta_1 = 10 \mu m$

else $\delta_1 = \delta_{max}$

	$\theta_i \leq 60^\circ$	$\theta_i > 60^\circ$
ductile target	$\alpha_1 = \alpha_2 = 2.6 ; \lambda = 0.02$	$\alpha_1 = \alpha_2 = 2 ; \lambda = 0.05$
brittle target	$\alpha_1 = 2.6 ; \alpha_2 = 3.5 ; \lambda = 0.1$	$\alpha_1 = \alpha_2 = 2 ; \lambda = 0.5$

Zenith density

$$g_{cone}(\theta) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(\theta - \theta_{max})^2}{2\sigma^2}\right) \mathbf{1}[0, 2\pi]$$

for $\theta_i \leq 60^\circ$: $\theta_{max} = \frac{\theta_{max 60} - \theta_{max 0}}{\pi/3} \theta_i + \theta_{max 0}$ radian units

for $\theta_i > 60^\circ$: $\theta_{max} = \theta_{max 60}$

$$\theta_{max 60} = 80^\circ$$

$$\theta_{max 0} = 30^\circ$$

$$\sigma = 3^\circ$$

Azimuth density

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$$\text{for } \theta_i \leq 60^\circ: \quad h_{cone}(\varphi) = \frac{1}{2\pi} \left(\frac{3\theta_i}{2\pi - 3\theta_i} \cos(\varphi - \varphi_i) + 1 \right) \mathbf{1}_{[0, 2\pi]}$$

$$\text{for } \theta_i > 60^\circ: \quad h_{cone}(\varphi) = \frac{1}{\sigma' \sqrt{2\pi}} \exp\left(-\frac{(\varphi - \varphi_{max})^2}{2\sigma'}\right) \mathbf{1}_{[0, 2\pi]}$$

$$\sigma' = 5^\circ$$

Normalisation

$$M_{cone} = \int_{\delta} \int_{\theta} \int_{\varphi} \int_V \frac{\pi \rho_t \delta^3}{6} n_{cone}(\delta, \theta, \varphi, V) d\delta d\theta \sin\theta d\varphi dV$$

By developing $\sin\theta$ at order 0 (i.e.: $\sin\theta = \sin\theta_{max}$), we obtain:

$$K_{cone} = \frac{6 M_{cone}}{\pi \rho_t} \frac{(4 - \alpha 1)(4 - \alpha 2) (\delta_{max}^{1-\alpha 2} \delta_1^{\alpha 2} - \delta_{min}^{1-\alpha 1} \delta_1^{\alpha 1})}{(4 - \alpha 1)(1 - \alpha 2) \delta_{max}^{4-\alpha 2} \delta_1^{\alpha 2} + 3(\alpha 2 - \alpha 1) \delta_1^4 - (4 - \alpha 2)(1 - \alpha 1) \delta_{min}^{4-\alpha 1} \delta_1^{\alpha 1}} \frac{1}{\sin\theta_{max}}$$

Velocity

There is an inverse proportionality between diameter and velocity of ejecta:

$$V(\delta, \theta, \varphi) = D(\varphi)/\delta + E(\varphi)$$

$$D(\varphi) = \frac{V_{max}(\varphi) - V_{min}}{\delta_{max} - \delta_{min}} \delta_{max} \delta_{min}$$

$$E(\varphi) = \frac{V_{min} \delta_{max} - V_{max}(\varphi) \delta_{min}}{\delta_{max} - \delta_{min}}$$

$$V_{min} = 10 \text{ m/s}$$

$$\text{for } \theta_i \leq 60^\circ: \quad V_{max}(\varphi) = \frac{V_i}{\cos\theta_i} \left(1 - \frac{3\theta_i}{4\pi} (1 - \cos(\varphi - \varphi_i)) \right)$$

$$\text{for } \theta_i > 60^\circ: \quad V_{max}(\varphi) = 3 V_i$$

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4.2.2.1.4 Spallation Process Modelling

No spallation is considered on ductile targets.

We propose a formulation with separated variables:

$$n_{spalls}(\delta, \theta, \varphi, V) = K_{spalls} f_{spalls}(\delta) g_{spalls}(\theta) h_{spalls}(\varphi) j_{spalls}(V)$$

Size distribution

We assume that all the spall particles have the same mass. Spalls have a plate-like shape, so we define an equivalent diameter:

$$\delta_{spalls} = \sqrt[3]{\frac{6 M_{spalls}}{N_{spalls} \pi \rho_t}}$$

We suggest: $N_{spalls} = 20$

$$f_{spalls}(\delta) = \Delta(\delta - \delta_{spalls})$$

Zenith distribution

$$g_{spalls}(\theta) = \frac{1}{\theta_{spalls}} \mathbf{1}[0, \theta_{spalls}]$$

$$\theta_{spalls} = 5^\circ$$

Azimuth distribution

$$h_{spalls}(\varphi) = \frac{1}{2\pi} \mathbf{1}[0, 2\pi]$$

Velocity distribution

$$V_{spalls} = 10 \text{ m/s}$$

$$j_{spalls}(V) = \Delta(V - V_{spalls})$$

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Normalisation

$$N_{spalls} = \int \int \int \int n_{spalls}(\delta, \theta, \varphi, V) d\delta d\theta \sin\theta d\varphi dV$$

$$K_{spalls} = N_{spalls} \frac{\theta_{spalls}}{1 - \cos\theta_{spalls}}$$

4.2.2.2 Model Inputs, Parameters and Outputs

This section summarises all inputs, parameters, and outputs used in the ejecta model.

Model inputs:

- particle density (ρ_p)
(constant, or as specified in the sections 2.1.2.3 and 2.2.8.1)
- particle mass (m_p)
(within the particle mass range to be considered according to the debris or meteoroid model mass distribution)
- target density (ρ_t)
(user input for ESABASE2/Debris, s. I25I, I26I)
- magnitude of the impact velocity (v_i)
(meteoroid or debris model output)
- incidence of the impact direction with respect to the surface (θ_i)
(determined from meteoroid or debris model output)
- azimuth of the impact direction (φ_i)
(Meteoroid or debris model output)
- type of target (brittle) or (ductile)
(determined by ESABASE2/Debris based on the selected damage size equation k_c -factor: if $k_c > 5 \rightarrow$ brittle target, else ductile target)
- number of directions for the modelling of the debris cone (N_{cone})
(user input)

Model outputs:

The ray-tracing technique is used for debris cone and spallation phenomena.

For each ray (i), outputs are:

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- azimuth (φ_i),
 - elevation (θ_i),
- concerning debris cone*
- size of the fragments (δ_i) in width interval ($\Delta\delta_i$),
 - number of fragments (n_i) of size (δ_i),
 - velocity of fragments (v_i) of size (δ_i),

concerning spallation

- fragment mass (m_{spall}),
- fragment velocity (v_{spall}) .

The model parameters and limit angles are described in the notes Ref. /16/, /17/, /24/.

4.2.2.3 Software Limits

This model should be used under following conditions:

- projectile diameter between 1 μm and 1 mm,
- impact velocity between 1 km/s and 20 km/s,
- thick target ¹⁾,
- ductile and brittle homogenous targets.

Some uncertainties remain concerning the size and velocity of ejecta fragments. Results are in fact very different from one author to another, as they used different measurement techniques. A direct relation between fragment size and its velocity is proposed.

Notes: 1) In the software implementation, the ejecta model is only available for objects with single wall design.

4.2.3 Conclusion

A complete improved ejecta model is implemented, based on the review of existing publications on various experiments and on theoretical considerations. Normal and oblique impacts are taken into account.

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Different phases of projectile impacts are studied: jetting phase, debris cone phase and spallation phase.

The model inputs are the projectile and target properties and the impact characteristics. The output is the ejecta distribution in the geometrical space around the impact by solid angle unity, in term of fragment size and velocity.

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5 The Impact and Damage Probability Analysis

This chapter describes the methodology of the probability analysis for impacts and damage which is implemented in the ESABASE2/Debris analysis tool. It also briefly describes the software tool. For more details on the tool itself and its usage, refer to the software user manual (Ref. /14/).

5.1 General

The analysis of the micro-particle risk is based on the integration of the impact probabilities delivered by the space debris and micro meteoroid models over area and time of the spacecraft mesh. The spacecraft mesh and object orientation is delivered by the ESABASE2 framework, defined in the *.geometry* file. The spacecraft orbit is generated by the ESABASE2 SAPRE module, which together with the mesh and kinematics define spacecraft velocity and orbital position, as well as the orientation and all spacecraft model elements at each orbital point of the analysis. For more details on the modelling in ESABASE2, the user is invited to consult the general ESABASE and ESABASE2 documentation.

5.2 The Weighted Ray-tracing Method

The integration of the impacts probabilities is performed using the flux models and a Monte Carlo raytracing method available in the ESABASE2 framework. The raytracing analysis method consists of the following main steps:

For each ray:

- 1) Generation of the micro particle impact velocity vector V .
- 2) Generation of a random impact point P on the element.
- 3) Generation of an emitting ray, launched from the point P and in the opposite direction of V .
- 4) Check if the emitted ray reaches free space; if so, a particle from this direction may hit the element and the ray is retained for further processing.
- 5) Computation of the damage related entities (ballistic limit, crater / hole diameter).
- 6) Computation of the weighted probabilistic data for the ray (see below)

For the summed results of all rays

- 7) Computation of flux, fluence and total quantity of impact and damage data
- 8) Computation of averaged impact data (impact velocity and angle, Ks factor [see below])

The weighted ray tracing technique method is based on the following facts:

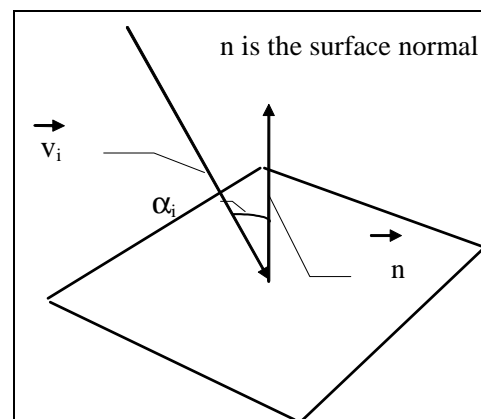
- The particle flux from a certain direction is proportional to the projection of the element surface onto a plane perpendicular to this direction, i.e. to the cosine of the impact angle α .
- For fluxes related to particle velocities, the flux is proportional to the ratio between impact velocity and particle velocity; this is the case for meteoroids.
- The element data is obtained by dividing the summed weighted ray data with the number of emitted rays¹, corrected with a factor depending on the ray flux model; for flux models generating fluxes on a random tumbling plate from a spherical direction generation, this factor is 4.

For the surface, the flux computation is expressed by the following equation:

$$FLUX = \int_{m_{min}}^{m_{max}} \int_{sphere} Flux(m) \cdot (v_i \cos \alpha) \cdot d\alpha \cdot dm$$

On ray level, this is equivalent to the summing up of the individual ray data.

$$DATA = \frac{\sum_{i=1}^{n_{ray}} F_s \cdot data_{ray-i} \cdot \cos \alpha_i}{n_{ray}}$$



F_s is the shading factor: $F_s = 1$ no shading,

$F_s = 0$: ray totally shaded. F_s is delivered by the ray tracing

The above described raytracing method automatically generates the k factor of the surface element (which depends on element orientation and spacecraft velocity [for meteoroids]); this is described in detail in annex C of ref. (Ref. /15/). The Earth shielding is

¹ The sum of the rays contains the weighting factor from the point seeding process of the raytracing library.

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treated as a surface of the geometric model: if the emitted ray lies within the Earth cone, it is considered as shaded.

The following results are computed and summed for all non-shaded rays:

- $\cos \alpha$ cosine impact angle
- $\alpha (\cos \alpha)$ impact angle times cosine impact angle
- $v_i (\cos \alpha)$ impact velocity times cosine impact angle
- $F_{flx} (\cos \alpha)$ Failure flux times cosine impact angle
- $C_{rflx} (\cos \alpha)$ Crater flux times cosine impact angle
- $Flx (\cos \alpha)$ Impact flux times cosine impact angle

The flux related data is computed with the difference of the flux of the considered particle size minus the flux of the maximum particle size of the analysis (the limit particle sizes are user input). This is a consequence of the cumulated flux formulation of the environment models, i.e. a flux for a given size s is given as the number of impacts of particles of equal or larger size than s per year and m^2 .

For impact fluxes, the minimum particle size is used. For ballistic limits, the critical or limit particle size (computed for the impact velocity vector with the ballistic limit equation). For cratered areas, a loop over particle size with the associated impact flux is run over the damage size equation and the data summed up.

5.3 Generation of Micro-Particle Impact Velocities

All the geometric features of the models, described in chapter 2, are simulated with raytracing. An essential part of the method relies on the proper generation of particle arrival directions. Depending on the particle type and model, a different type of particle generation is applied. The different methods implemented in the software and their applications are briefly explained in the following sections.

5.3.1 Space Debris Particle Velocity Generation

Basically two impact direction schemes are used in the software: 1 for the NASA 90 model and one for the MASTER 2001, MASTER 2005 and MEM models. The difference relies on the fact that the NASA 90 model impact direction generation is performed in the plane normal to the Earth direction, the MASTER 2001 MASTER 2005 and MEM impact direction generation in 3D space.

Both impact direction schemes rely on the use of cumulated direction probabilities and a random number generator. The scheme is visualised in the example below.

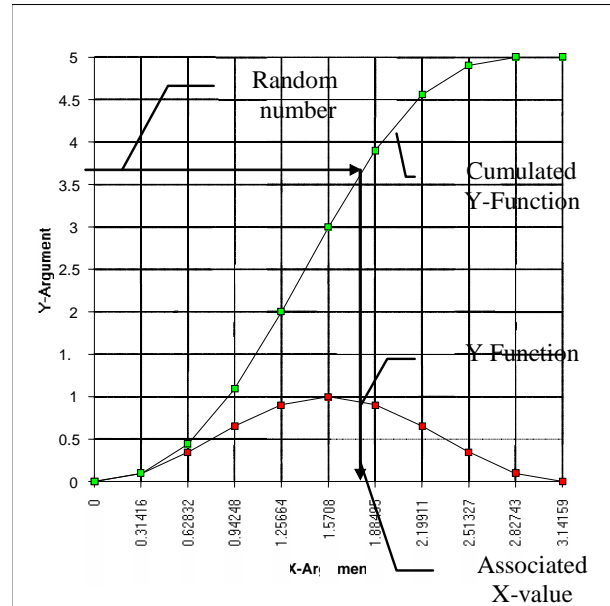
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For a distribution function, the cumulated function is generated.

A random number is generated between $Y(X = X_{min})$ and $Y(X = X_{max})$, in the case of the example between 0 and 5.

For the randomly generated value of Y -cumulated, the corresponding X -value is extracted: this is the sought value.

In the case of impact velocity, the direction and amplitude is thus generated from the velocity distribution function(s) of the space debris environment model.



For the NASA 90 model, the cumulated velocity amplitude and arrival angle are function of the orbit inclination. In the NASA 90 model, the debris orbits are assumed circular, thus the altitude dependent debris velocity component is given directly by the spacecraft velocity. Two random number generator calls are used for the impact velocity vector generation.

For the ORDEM 2000, the MASTER models and MEM, the flux vs. elevation and azimuth data and impact velocity is computed by the corresponding model with the spacecraft orbit and a sphere as spacecraft. The results of this analysis are stored on scratch files in case of ORDEM 2000, MEM and MASTER 2005 or provided via *COMMON* in case of MASTER 2001.

For the space debris impact velocity, the data is tabulated. For the MASTER models two independent data sets are created: the flux vs. elevation angle and the flux vs. azimuth angle. Associated with the azimuth angle is the impact velocity. Two random number generator calls are used to extract the impact velocity vector (amplitude, azimuth angle, elevation angle).

For MEM only one random number generator call is used to extract the impact velocity vector (amplitude, azimuth angle, elevation angle). With found elevation from cumulated 2D spectrum corresponding azimuth and velocity can be determined from the multidimensional MEM output.

5.3.2 Micro-Meteoroid Particle Velocity Generation

Contrary to the space debris, the micro meteoroid velocities are generated relatively to Earth. Two different schemes are used, depending on the meteoroid model used.

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Grün sporadic omni-directional model

In this model, the meteoroids may arrive from any direction. This random arrival is generated with two random number generator calls:

- For the azimuth angle between 0 and 2π
- For the cosine of the elevation angle between -1 and 1.

The velocity amplitude is generated from the chosen velocity distribution scheme:

- For constant velocity, the input amplitude is used
- For the NASA 90 velocity distribution, the amplitude is extracted from the distribution function with a random number generator call.
- For the Taylor HRMP, the velocity / flux function is used to generate the velocity amplitude

For more details, see chapter 2.

Meteoroid streams and β particles

For these enhanced options, the meteoroid velocity vector (arrival direction and velocity amplitude) is derived from the meteoroid enhancement model itself, see chapter 2. In the case these options are activated, the meteoroid ray is split into different classes:

- The main Grün portion ray, derived as described above
- In case the α/β -separation is activated, the Grün ray is split into the α part (of random direction) and the β with a fixed direction from the sun.
- In case the streams / interstellar source option is activated, each stream or interstellar source is provided with a ray of the corresponding velocity vector.

As can be seen, the enhanced meteoroid options may lead to noticeable increases in computing time. If the Grün model is switched off, the analysis can be performed with a smaller number of rays (50 is generally enough).

For both meteoroid velocity generation methods, the impact velocity is computed as a vector sum of the meteoroid velocity and the spacecraft velocity.

5.3.3 Implementation of the Stream & Interstellar Contribution

The stream velocity vector is extracted from the streams file for the specific time corresponding to the orbital step being analysed. The relative flux contribution is checked

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according to the scheme lined out in section 2.2.5.2. For missions which are longer than one orbit, the flux contribution is checked for all calendar times corresponding to the orbital point over the mission.

E.g. for a 10 day mission on a geostationary orbit, 10 calls to the stream file extraction routine will be performed, and the stream contribution of all active streams averaged for all these time steps. Depending on the mission duration, some streams may only be active for certain time steps. The number of calls to the stream extraction routine is echoed during analysis execution.

With this scheme, it is obvious that the directional information from the stream contribution will be lost for long mission durations. Also, for low Earth orbits, a large number of calls to the stream extraction routine will be performed for extended mission times (160 calls for an orbit with 90 minutes period and 10 days mission time).

The active stream numbers are echoed to the analyser listing file. The summed stream contribution over mission time is stored in the model view file, see ref. /14/.

The interstellar sources are treated as streams for velocity vector generation and flux contribution.

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5.4 Damage and Impact Probability Computations

On ray level, the impact, failure and crater fluxes are computed as follows for unshaded rays:

Impact flux:

$$i_{flx} = [flux(s_{min}) - flux(s_{max})] \cdot \cos \alpha$$

s is the impactor size, diameter or mass.

Failure Flux:

$$f_{flx} = [flux(s_{crit}) - flux(s_{max})] \cdot \cos \alpha$$

s_{crit} is delivered by the damage equation.

The crater flux is obtained by looping over the impactor size bins, summing the binned fluxes (flux of bin minimum size - flux of bin maximum size) multiplied with the crater / hole size produced by an impactor of logarithmic mean size of the bin.

Impact angle: $\alpha_i = i_{flx} \cdot \alpha \cdot \cos \alpha$

Impact velocity: $v_{imp,i} = i_{flx} \cdot v_{imp} \cdot \cos \alpha$

The impact and damage fluxes are computed with the following equations of the ray data:

Impact Flux: $I_{flx} = \frac{4 \sum \zeta \cdot i_{flx}}{n_{ray}}$ n_{ray} is the total number of rays fired per element.

$\zeta=1$ for non-shaded ray, $\zeta=0$ for shaded ray.

Failure Flux: $F_{flx} = \frac{4 \sum \zeta \cdot f_{flx}}{n_{ray}}$ The crater flux is obtained similarly.

Fluence and number of impacts / failures are computed from the flux data:

$Fluence = Flux \times Time$ Orbital arc: time of one revolution;

Mission: mission time

$Nb. \text{ of (impacts / failures / craters) } = Fluence \times Element \text{ area}$

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The probability of no failure is extracted from the number of failures with the following equation:

$$P_{no_failure} = \exp[-N_{failures}] \quad N_{failures} \text{ is the number of failures.}$$

The probability of no failures is computed on object and spacecraft level.

Additionally, the average impact velocity data and the Ks factor are computed as follows:

Average impact angle: $\bar{\alpha} = \frac{\sum \alpha_i}{\sum i_{flx}}$ Only non-shaded rays are processed.

Average impact velocity: $\bar{v}_{imp} = \frac{\sum v_{imp}}{\sum i_{flx}}$ Only non-shaded rays are processed.

The Ks factor: $Ks = \frac{4 \sum \zeta \cdot \cos \alpha_i}{n_{ray}}$

Note

The Ks factor is the combined value of the k factor (i.e. the impact flux ratio of the element to the flux of a random tumbling plate [for meteoroids: a fixed random tumbling plate, i.e. with $V_{sc} = 0$]) and the shading of the plate of Earth and neighbouring surfaces of the spacecraft.

Thus for a simple box, the Ks factor of the faces correspond to the k factor. This type of result can also be obtained with the fixed plate option, see ref. (Ref. /14/).

The above data is computed for each orbital point. The result is averaged for the orbital arc level output, which is appropriate for the flux computation over an orbit. The mission level results are simply the orbital arc flux results multiplied by the mission time.

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Annex A Particle Flux on Orbiting Structures

A.1 Introduction

Two factors interfere with the particle impact flux computation on an orbiting spacecraft, namely:

- 1) The influence of the spacecraft velocity on the impacting flux, referred to as the „k“ or “f_t“ factor in several papers, and
- 2) The Earth shielding of the omni-directional particle flux.

The presented results are implemented in the upgraded ESABASE2/DEBRIS software.

It is assumed that the particle flux is omni-directional, i.e. no direction is preferential. This corresponds to the Grün sporadic meteoroid flux model.

The influence of the spacecraft velocity will first be investigated for two factors:

- 1) The “f_t“ factor describing the relation between the flux on a moving oriented surface element and the flux on a virtual stationary surface element.
- 2) The “k_f“ factor describing the relation between the flux on the forward side of a moving, oriented surface (or plate) to the average flux on the surface (average = 0.5·[forward + lee fluxes]).

As will be seen, these two factors are linked and depend on the orientation of the surface element. Drawn from the above two factors, two additional factors can be defined:

- 3) The “f_t³“ factor, which is obtained by evaluating the f_t factor on three perpendicular planes, which corresponds to the ratio between the flux on a moving “random tumbling” surface element and the flux on a virtual stationary surface element. This factor can also be used for the total flux increase on a symmetric spacecraft.
- 4) The “k” factor, which is the ratio between the flux on the forward side of a surface element (or plate) to the flux on a virtual stationary randomly oriented surface element.

A.2 Theoretical Description of the Particle Impact Flux on a Moving Plate

A.2.1 General Description

For the general case, we need to consider an omni-directional particle flux and an arbitrary direction of motion of the plate.

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Let us first consider the situation depicted in fig. A-1 of a particle hitting the moving plate under an angle α_i . The plate is moving with a velocity \vec{v}_m . \vec{v}_s makes an angle β with the surface normal vector \vec{s} .

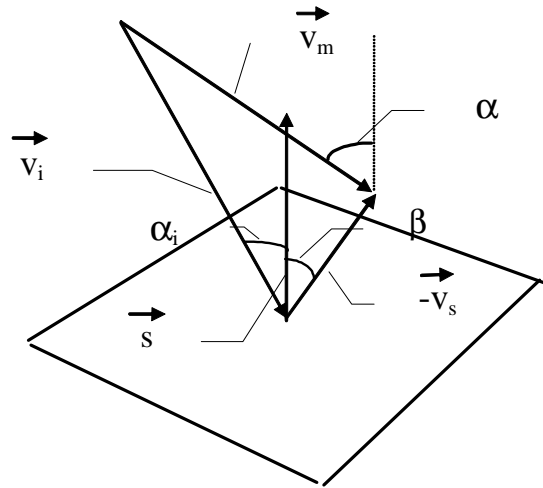


Figure A-1 Oblique flux on a plate moving in an arbitrary direction

The impacting flux from the particles impinging with the velocity \vec{v}_i on the plate is obtained by the product between the particle probability, the scalar impact velocity and the cosine of the angle between impact velocity and surface normal. The latter can be described by the scalar product between the impact velocity \vec{v}_i and the surface element normal vector \vec{s} :

$i = n(v_m) \cdot \vec{s} \cdot \vec{v}_i$ where $n(v_m)$ is the probability of a particle arriving with a velocity v_m , taken from the velocity distribution.

The impact velocity can be expressed as: $\vec{v}_i = \vec{v}_m + \vec{v}_s$, where \vec{v}_s is chosen positive in the direction towards the plate.

Using the distributivity of the vector sum with respect to the scalar product, we obtain:

$$i = n(v_m) \cdot \vec{s} \cdot (\vec{v}_m + \vec{v}_s) = n(v_m) \cdot (\vec{s} \cdot \vec{v}_m + \vec{s} \cdot \vec{v}_s) = n(v_m) \cdot s \cdot (v_m \cos \alpha + v_s \cos \beta) \quad (9)$$

In order to assess the complete situation, we must evaluate the integration limits of the particle angle α of the whole „captured“ spherical portion seen by the plate. The situation is best illustrated with a 2D sketch, see fig. A-2 below.

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$$di = i(v_m, \alpha) \cdot ds = n(v_m) \cdot (\vec{v}_i \cdot \vec{s}) \cdot 2\pi \sin \alpha \cdot d\alpha \cdot dv_m$$

$$di = 2\pi \cdot n(v_m) (v_m \cos \alpha + v_s \cos \beta) \sin \alpha \cdot d\alpha \cdot dv_m$$

$2\pi \sin \alpha \cdot d\alpha$ is the surface integration argument computed from the integration over φ of the spherical surface differential $\sin \alpha \cdot d\varphi \cdot d\alpha$.

We thus obtain the following integral:

$$I = \int_0^\infty \int_0^{\alpha_{lim}} 2\pi \cdot n(v_m) \cdot (v_m \cos \alpha + v_s \cos \beta) \cdot \sin \alpha \cdot d\alpha \cdot dv_m \quad (11)$$

α_{lim} is obtained from equation (17).

A.2.2 Particle Flux on a Plate with Unique Particle Velocity

We shall first consider the situation where the particles have a unique velocity.

With a unique particle velocity, equation (18) boils down to a single integral:

$$I = 2\pi \cdot n_0 \int_0^{\alpha_{lim}} (v_m \cos \alpha + v_s \cos \beta) \cdot \sin \alpha \cdot d\alpha$$

with n_0 being the probability of a particle arriving from a random direction. This integral can easily be solved analytically:

$$I = 2\pi \cdot n_0 \int_0^{\alpha_{lim}} (v_m \cos \alpha + v_s \cos \beta) \cdot \sin \alpha \cdot d\alpha$$

Substituting and solving the equations, we finally obtain:

$$I = \frac{\pi \cdot n_0}{v_m} (v_m + v_s \cos \beta)^2$$

The above expression for I permits the analytical computation of the k and f_t factors. The f_t factor needs the total flux on an orbiting structure:

For an oriented plate:
$$I_{tot} = \frac{\pi \cdot n_0}{v_m} [(v_m + v_s \cos \beta)^2 + (v_m - v_s \cos \beta)^2]$$

For the case with $v_s = 0$: $I_{tot} = 2\pi n_0 v_m$.

We can now derive the analytical expressions of the two factors:

$$k_f = \frac{2 \cdot I(+v_s)}{I(+v_s) + I(-v_s)} \quad (12)$$

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$$f_t = \frac{I_{tot}(v_s)}{I_{tot}(v_s=0)} \quad (13)$$

A spacecraft can be symbolised by a regular box, moving with one side normal to the flight direction, i.e. two sides with $\beta = 0$ and four sides with $\beta = 90^\circ$. This of course implies a symmetrical spacecraft structure. We can now derive the f_t^t factor for the whole spacecraft:

$$f_t^t = \frac{1}{6v_m^2} [4v_m^2 + (v_m + v_s)^2 + (v_m - v_s)^2] = \frac{6v_m^2 + 2v_s^2}{6v_m^2}$$

We finally obtain:

$$f_t^t = 1 + \frac{v_s^2}{3v_m^2} \quad (14)$$

In the same way one can compute the k factor:

$$k = \frac{I(+v_s)}{I(v_s=0)} = \frac{2 \cdot I(+v_s)}{I(+v_s) + I(-v_s)} \cdot \frac{I(+v_s) + I(-v_s)}{2 \cdot I(v_s=0)} = f_t \cdot k_f \quad (15)$$

Tables A-1 and A-2 below show the f_t , k_f and k factors for two velocity values and a set of values of β .

$$I^{*+} = \frac{I(+v_s)}{\pi v_0 v_m}$$

$$I_{tot}^* = \frac{I_{tot}(+v_s)}{\pi v_0 v_m}$$

Case 1: $v_s = v_m$

β [deg]	0	15	30	45	60	75	90
I^{*+}	4	3.86	3.48	2.91	2.25	1.58	1
I_{tot}^*	4	3.87	3.5	3.0	2.5	2.13	2
k_f	2.0	1.99	1.99	1.94	1.8	1.49	1.0
k	4.0	3.84	3.48	2.91	2.25	1.59	1.0
f_t	2.0	1.93	1.75	1.5	1.25	1.07	1.0

Table A-1 Analytical values for k and f_t with $v_s = v_m$

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$$f_t^t = 1.333$$

Case 2: $v_s = 7.6$ km/s; $v_m = 16.8$ km/s

β [deg]	0	15	30	45	60	75	90
I^{*+}	2.11	2.06	1.94	1.74	1.5	1.25	1
I_{tot}^*	2.41	2.38	2.31	2.2	2.1	2.03	2
k_f	1.75	1.73	1.68	1.58	1.43	1.23	1.0
k	2.1	2.06	1.93	1.74	1.5	1.24	1.0
f_t	1.2	1.19	1.15	1.1	1.05	1.01	1.0

Table A-2 Analytical values for k and f_t with $v_s = 7.6$ km/s; $v_m = 16.8$ km/s

$$f_t^t = 1.068$$

A.2.3 Particle Flux on a Plate with a given Velocity Distribution

The general expression of I is given by equation (18) in paragraph A.2.1. Proceeding as in the previous section, we can solve the first integration step (over α). We now have the following equation:

$$I = \int_0^{\infty} \frac{\pi}{v_m} n(v_m) \cdot (v_m + v_s \cos \beta)^2 dv_m \quad (16)$$

In order to proceed further, we need to define the form of the velocity distribution $n(v_m)$.

An easy way to approach the velocity distribution is to define a function composed of a series of straight curves as depicted in figure A-3. Actually most velocity distributions can be approximated with such a function.

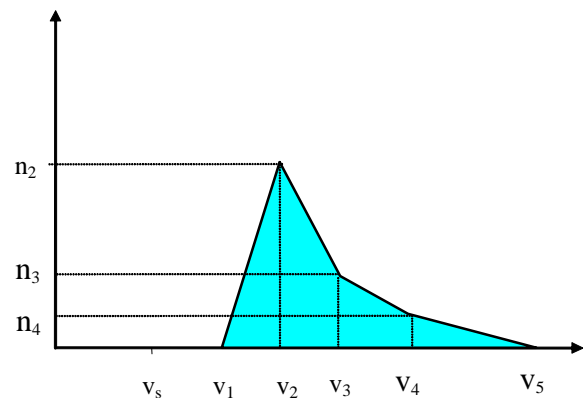


Figure A-3 Generalised velocity distribution

To derive the general formulation for I , let's integrate the portion of the velocity function between v_1 and v_2 :

$$v_1 < v_m < v_2 \quad n(v_m) = n_2 + (n_1 - n_2) \frac{v_2 - v_m}{v_2 - v_1}$$

In fig. A-3, one can split the integration

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into four steps.

An integration step I_2 between v_1 and v_2 becomes:

$$I_2 = \pi \int_{v_1}^{v_2} \left(n_2 + \frac{(n_1 - n_2)(v_2 - v_m)}{v_2 - v_1} \right) \frac{(v_m + v_s \cos \beta)^2}{v_m} \cdot dv_m$$

The above expression can be analytically solved. This process is described in (Ref /15/). One finally obtains:

$$I = \pi \sum_{i=2}^m \left\{ \frac{2n_i + n_{i-1}}{6} v_i^2 - \frac{2n_{i-1} + n_i}{6} v_{i-1}^2 + \frac{n_{i-1} - n_i}{6} v_i v_{i-1} + (n_{i-1} + n_i) v_s \cos \beta (v_i - v_{i-1}) \right. \\ \left. + (n_i - n_{i-1}) v_s^2 \cos^2 \beta + \frac{n_{i-1} v_i - n_i v_{i-1}}{v_i - v_{i-1}} v_s^2 \cos^2 \beta \cdot \ln \left(\frac{v_i}{v_{i-1}} \right) \right\} \quad (17)$$

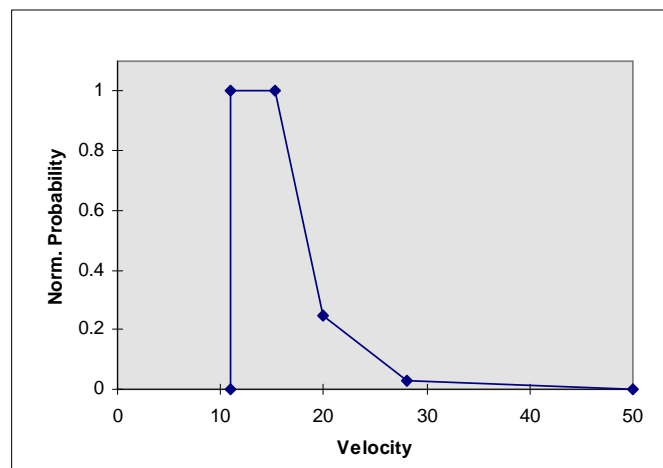
Equation (24) above is easily programmable. The two standard meteoroid velocity distributions, the NASA 90 model and the Cour-Palais model were approximated with 5 ($n_i; v_m$) points and computed with equation (24). The results are presented in tables A-3 and A-4.

For comparison with the data in chapters A.2.2 and A.3, the plate velocity was set at 7.6 km/s.

Analytical flux computation of the NASA 90 model

The NASA 90 velocity distribution can be approximated as follows:

Velocity	Normalised Probability
11.0	1.0
15.2	1.0
20.0	0.25
28.0	0.03
50.0	0.0



The analysis over the above velocity distribution with equation (24) gives the following results:

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	$v_s = 0$	$v_s = 7.6$						
β [deg]	N/A	0	15	30	45	60	75	90
Flux positive	451.6	966	945.1	885.3	794.3	683.5	565.3	451.6
Flux total	903.2	1106	1092	1055	1005	953.8	916.8	903.2
k_f factor	1.0	1.75	1.73	1.68	1.58	1.43	1.23	1.0
k factor	N/A	2.14	2.09	1.97	1.75	1.52	1.25	1.0
f_t factor	N/A	1.22	1.21	1.17	1.11	1.06	1.015	1.0

Table A-3 Results with $v_s = 7.6$, NASA 90 Velocity distribution

The f_t^{\dagger} factor amounts to 1.075.

Comparing table A-3 to table A-2, the results are very close.

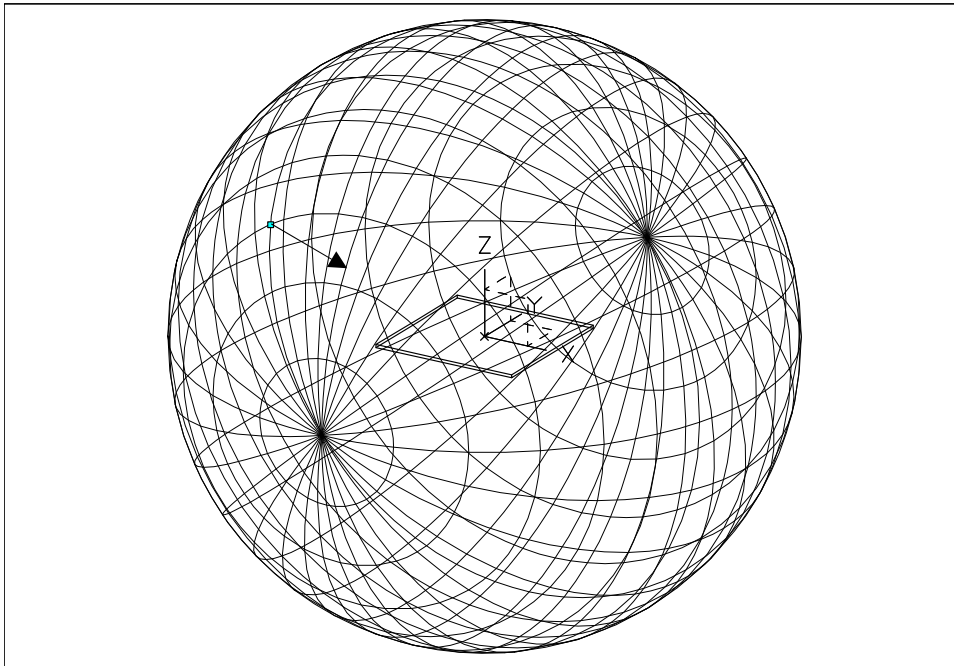
A.3 Validation of the numerical Approach with Ray Tracing

A.3.1 General

In order to assess the behaviour of the ray tracing technique implemented in ESABASE and also to double-check the analytical derivations of chapter A.2, two computer programs were written to numerically simulate the effects of an omni-directional particle flux environment on a moving plate:

- The first program simulated the true environment, firing rays randomly from a sphere onto a plate

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- The second program corresponds to the raytracing technique implemented in the enhanced software, firing the rays from the plate and weighting the ray data with the cosine of the impact angle.

The ray hits were recorded and the factors computed analytically in the previous section evaluated. The results are shown in the following two subsections.

The ray hits were counted on both sides of the plate. The case for a box was derived from the single plate results.

A.3.2 Results when the Rays are fired from a Unit Sphere

The program K_SPHERE was run with 2.0E6 rays, with two velocity configurations. The first is the extreme case where the plate velocity equals the particle velocity, the second the case treated in the previous chapter, i.e. with 16.8 particle velocity and 7.6 plate velocity. The ray scaling factor is 0.001.

The results are presented in tables A-5 and A-6.

Case 1: $v_s = v_m = 9.25$

	$v_s = 0$	$v_s = 9.25$						
β [deg]	N/A	0	15	30	45	60	75	90
Nhit positive	12911	34090	32996	29836	25839	20468	15577	10898
Nhit negative	13050	0	68	489	1679	3702	6836	10737

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	$v_s = 0$	$v_s = 9.25$						
β [deg]	N/A	0	15	30	45	60	75	90
Nhit total	25961	34090	33064	30325	27518	24170	22413	21635
Flux positive	119.4	473.9	457.1	409.6	346.0	262.7	186.6	119.3
Flux negative	120.7	0	0.15	2.1	10.1	29.4	65.3	116.8
Flux total	240.1	473.9	457.2	411.8	356.1	292.1	251.9	236.1
Flux total 3 plates	717.3	945.0	943.2	938.8	950.1	938.8	943.2	946.3
Imp. angle positive	44.9	38.0	34.5	36.6	40.2	44.9	50.3	51.9
Imp. angle negative	45.0	--	84.5	79.3	73.5	67.9	62.1	51.6
k_f factor	0.995	2.0	2.0	1.99	1.94	1.8	1.48	1.01
f_t factor	N/A	1.97	1.9	1.72	1.48	1.22	1.05	0.98
k factor	N/A	3.94	3.8	3.42	2.87	2.2	1.55	0.99
f_t^t factor	N/A	1.32	1.32	1.31	1.33	1.31	1.32	1.32

Table A-5 Results from K_SPHERE with $v_s = v_m = 9.25$

Case 2: $v_s = 7.6$; $v_m = 16.8$

	$v_s = 0$	$v_s = 7.6$						
β [deg]	N/A	0	15	30	45	60	75	90
Nhit positive	12893	21278	21071	20071	18555	16835	14563	12510
Nhit negative	12787	5855	6091	6567	7544	8890	10719	12567
Nhit total	25680	27133	27162	16638	26099	25725	25282	25077
Flux positive	216.6	455.6	448.5	420.0	377.5	328.5	269.1	215.8
Flux negative	214.8	65.5	69.5	80.2	99.8	129.1	170.7	216.6
Flux total	431.4	521.1	518.0	500.2	477.2	457.6	439.7	432.4
Flux total 3 plates	1301	1386	1389	1390	1387	1390	1389	1386
Imp. angle positive	44.9	39.4	39.8	40.7	41.9	43.5	45.3	47.2
Imp. angle negative	45.3	52.5	52.0	52.4	51.6	50.5	49.3	47.4
k_f factor	1.004	1.75	1.73	1.68	1.58	1.44	1.22	1.0

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	$v_s = 0$	$v_s = 7.6$						
β [deg]	N/A	0	15	30	45	60	75	90
f_t factor	N/A	1.21	1.20	1.16	1.11	1.061	1.02	1.0
k factor	N/A	2.12	2.08	1.95	1.75	1.53	1.24	1.0
f_t^t factor	N/A	1.065	1.068	1.068	1.066	1.068	1.068	1.065

Table A-6 Results from K_SPHERE with $v_s = 7.6$ $v_m = 16.8$

A.3.3 Results when the Rays are fired from plate centre

The program K_PLATE simulates the ESABASE approach by firing particles from the plate (centre) in random directions. The impact flux is computed.

The program K_PLATE was run with 26000 rays and with two velocity configurations. The same cases are studied as in the previous section. The flux scaling factor is 0.001.

The results are presented in tables A-7 and A-8.

Case 1: $v_s = v_m = 9.25$

	$v_s = 0$	$v_s = 9.25$						
β [deg]	N/A	0	15	30	45	60	75	90
Nhit positive	12929	26000	25531	24291	22108	19563	16247	13107
Nhit negative	13071	0	469	1709	3892	6437	9753	12893
Nhit total	26000	26000	26000	26000	26000	26000	26000	26000
Flux positive	59.71	238.7	231.8	209.1	174.2	136.7	94.61	60.43
Flux negative	60.0	0	0.1	1.01	5.2	15.0	33.2	59.38
Flux total	119.7	238.7	231.9	210.2	179.5	151.7	127.8	119.8
Flux total 3 plates	361.1	478.8	479.9	481.9	481.6	481.9	479.9	478.8
Imp. angle positive	45.1	33.9	34.5	36.9	40.4	44.8	50.3	56.2
Imp. angle negative	45.4	--	84.6	79.1	73.7	67.8	62.2	56.0
k_f factor	1.0	2.0	2.0	1.99	1.94	1.8	1.48	1.01
f_t factor	N/A	1.99	1.94	1.76	1.5	1.27	1.07	1.0
k factor	N/A	3.98	3.88	3.5	2.91	2.29	1.58	1.01

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	$v_s = 0$	$v_s = 9.25$						
β [deg]	N/A	0	15	30	45	60	75	90
f_t^t factor	N/A	1.33	1.33	1.34	1.33	1.34	1.33	1.33

Table A-7 Results from K_PLATE with $v_s = v_t = 9.25$

Case 2: $v_s = 7.6$; $v_m = 16.8$

	$v_s = 0$	$v_s = 7.6$						
β [deg]	N/A	0	15	30	45	60	75	90
Nhit positive	12913	18894	18585	18054	17179	16027	14542	12979
Nhit negative	13087	7106	7415	7946	8821	9973	11458	13021
Nhit total	26000	26000	26000	26000	26000	26000	26000	26000
Flux positive	108.1	230.2	224.3	211.0	190.4	165.1	136.6	109.0
Flux negative	109.6	32.7	35.0	40.5	51.1	65.3	84.4	109.2
Flux total	217.7	292.8	259.3	251.5	241.4	230.4	221.0	218.2
Flux total 3 plates	654.9	700.3	698.5	700.0	699.9	700.0	698.5	700.3
Imp. angle positive	45.0	39.6	39.7	40.8	42.0	43.4	45.3	47.5
Imp. angle negative	45.0	52.8	52.1	52.4	51.2	50.5	49.3	47.3
k_f factor	0.99	1.75	1.73	1.68	1.58	1.43	1.24	1.0
f_t factor	N/A	1.21	1.19	1.16	1.11	1.058	1.02	1.0
k factor	N/A	2.12	2.06	1.95	1.75	1.51	1.26	1.0
f_t^t factor	N/A	1.069	1.067	1.069	1.069	1.069	1.067	1.069

Table A-8 Results from K_PLATE with $v_s = 7.6$ $v_m = 16.8$

A.4 Conclusions and Discussion

The numerical results presented in the previous sections prove full agreement between the k and f_t factors computed analytically or numerically.

The main conclusions are:

- For a plate, the flux is strongly dependent on the orientation of the plate with respect to the flight direction. Also the total flux (i.e. the f_t factor) depends on this. For a plate in

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LEO, the flux increase when the plate is normal to the flight direction compared to the flux when parallel to the flight direction amounts to 20%.

- The ray tracing technique used in ESABASE perfectly captures the true particle flux situation, provided that the $\cos\alpha$ term (angle between impact direction and surface normal) is introduced in the flux computation.

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